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The Professional Needs of Secondary School Teachers of Mathematics[1]

By DANIEL W. SNADER
University of Illinois, Division of Undergraduate Studies, Galesburg, Ill.

THERE are several understandable reasons for selecting this subject, "The Professional Needs of Secondary School Teachers of Mathematics," for my discussion with you this morning. I believe it is good practice for members of our profession to take a periodic inventory of the work we have accomplished and to see how nearly we have reached our professional objectives. It is essential that we fulfill this obligation with open minds, critically evaluating our achievements, determining our weaknesses and strengths, and finally planning a course of action which will make our teaching more dynamic, purposeful, and altogether more effective.

THE SITUATION IN RETROSPECT

Before we consider our professional needs, let us review hastily some of the major purposes of the American system of education and the part which mathematics is supposed to play in a truly functional curricular program. The typical American school system has been developed for the purpose of perpetuating and improving certain basic democratic ideals. Democracy as viewed through these ideals becomes a desirable pattern of living. It is the obligation of the schools to assist students in cultivating and controlling those skills, attitudes, and types of behavior which are necessary for democratic living.

The school program must provide those activities which will orient the students properly with the society in which they live. It must also provide the basic knowledge against which to project problem situations, to provide patterns of thought for analyzing every day situations, and to provide skills for controlling and adapting them. To be able to do and to be able to know should be regarded as of equal importance, for behavior is without proper guidance except as knowledge may be used to provide such guidance.

One of the highest democratic ideals which we have is that of self-realization, the concept of education which implies that our students be given the opportunity for achieving the greatest development of their individual abilities, and potentialities. It is regrettable in curriculum construction that development of the higher intellectual processes are so often obscured by an over-emphasis upon the development of the social aspects. The development of the higher mental processes are of

[1] Paper read at the University Workshop for Teachers of Mathematics. Dr. Snader was the director of the Workshop.

major importance to the individual who is capable of them and to the nation to which the individual belongs.

Professor Judd in his discussion of education as the cultivation of higher mental processes says, "The great rewards of civilization go not to the men who are strong of muscle or swift of foot, but to those who advance knowledge and elevate human attitudes. If by any means the educational system can discover how to promote, even in the slightest measure, the development of the higher mental processes, great achievements will be gained for civilization."

When a curriculum is designed to meet the goals of general, social, and individual competence, mathematics will have an important part in it. The concepts, the methods, and formulations at the elementary level are useful to individuals in many situations having social, economic and other practical implications. Mathematics provides a medium through which to understand and control natural and social phenomena and offers excellent opportunities at all levels for developing higher mental processes in the form of drawing correct generalizations and applying them once they are attained.

What Constitutes a Functional Program

The changes required to keep the program of the American public schools abreast of the rapid changes taking place in the social, political, and industrial life of the nation are observable through the changes that are taking place in the philosophy and the content of the public school curriculum.

During the course of recent years there have been many changes in the organization and presentation of subject matter at the elementary and secondary school level of instruction. In the junior high school these changes have been especially pronounced. There has been a deletion of many traditional practices and areas of subject matter, and the injection of new and more useful materials.

There has been much consideration given to the time and place for introducing certain concepts and skills, to the meaning and purpose of drill, and to a revitalization of instructional techniques. In the senior high school the changes in the mathematics program have been less pronounced. For the most part they have been confined to a readjustment of traditionally accepted units of instruction and the introduction of some new material. On this level there has been very little noticeable effort to part with tradition; in fact, there has been a "rather persistent acceptance of a convenient heritage of teachable units of subject matter, selected and organized largely in terms of a factual and logical pattern of instruction."

A great many recent proposals for curriculum reconstruction and reform have been an outgrowth of a new philosophy of education. We are all quite aware of this philosophy. It emphasizes desirable patterns of conduct and the development of personality rather than primarily the rigid disciplines of logically organized subject matter. It makes subject matter good or bad, depending upon whether it can be used effectively in the development of a better personality. When a curriculum is formulated upon the basis of this philosophy then the content is determined by its functional worth in the total instructional program which is designed to provide the student with varied opportunities "to understand, interpret, and appreciate the major functions of life as a member of a democratic social order."

Dr. Briggs in his "A Philosophy of Secondary Education Today" states, "The basic function of education is to make youth better disposed and better able to contribute to the betterment of society, either by participating with their maximum effectiveness in the accepted modes of life or by perceiving other and better modes, which they are active in convincing their fellows are superior."

If the teaching of mathematics is to make its optimum contribution to the attainment of such objectives under this

philosophy of education, then we as teachers must be aroused from the static satisfaction in the historic perfection of our subject and brought to realize the need for formulating its content into units of instruction that are vital, significant and timely.

How Does Mathematics Function in This Program

Most of us will agree that mathematical techniques offer very effective means of investigating, tabulating, classifying, and interpreting natural and social events. Mathematical concepts and symbolism present a means of concise expression which is simple and exact. Mathematical subject matter sets a pattern for logical precision of thought and for objective evaluation.

Most teachers of mathematics agree with the general educationist that "the aims of education for American schools must be defined in terms of certain generalized controls of conduct which, if developed, will lead to the realization of the democratic ideal." However, they hasten to point out that a great many of these "generalized controls of conduct" reach their maximum meaning and significance only through mathematical interpretation. Mathematics is vitally and inextricably bound with the culture of the world. An educational program formulated to meet the demands of the modern educational philosophy would be fundamentally unsound and quite incomplete if it should omit mathematics from its content. When mathematics is included in such a program, it provides vast resources of pertinent materials and an abundance of stimulating opportunities for developing powers of understanding and of analyzing space and quantity relationships which are so essential to an insight into and control over our present environment. It provides us with a ready source of abstract problem materials through which to develop habits of thought and action.

Our educational philosophy will, to a large extent, determine the nature of the content of the mathematical materials which are included in our secondary school curriculum. The techniques of instruction will be based upon the psychology of learning developed to implement the program. "The mathematical content of the curriculum should be so selected, organized, and presented that it will reveal to the child in the schoolroom and the layman on the street something of the essential significance of this language of algorism and abstraction as an interpreter of his immediate environment and the universe about him." This is a large order, when we think of the curriculum of our traditional schools in which the subject-matter is organized according to rather rigid patterns of logical sequence. The child under such circumstances is usually subjected to severe disciplines in accordance with a philosophy that defined education in terms of learning and book knowledge.

Today the pendulum has swung in the opposite direction and thus, under the pressure of naturalistic philosophy of education, we find a tendency to have the *child* replace *subject-matter* as the focal point in our educational program. Surely somewhere between these two extremes we should be able to find a common meeting ground in which we do not present mathematics solely for its disciplinary values, nor do we give way to the childish whims of our students under the guise of providing a program merely to satisfy their "felt needs." We would be "skating on thin ice" if we were to attempt to build a secondary school program entirely upon the so-called "felt needs" of our students. On the other hand, we would be derelict in our professional duty if we did not make an honest, and concerted attempt at planning at least a two-track mathematics program for students of our secondary schools.

During the past three years I had the pleasure of serving as a member of the New York State Mathematics Revision Committee working with the State Department of Education in revising the various existing syllabi in mathematics (grades 7 through 12) along the lines

suggested in the "Report of the Joint Committee of the Mathematical Association of America and The National Council of Teachers of Mathematics," (Published as the 15th yearbook of the National Council of Teachers of Mathematics.) I shall not go into a detailed discussion of this experience. However, since I represented the Teachers' Colleges of the state, my interest and responsibility in these attempts to produce unified programs in mathematics in grades 7 through 12, on a two-track plan of development, lay not only in the selection of the instructional content, and grade placement of topics, but also in the new conception and formulation of an adequate professional program for teachers of mathematics to do justice to the new curriculum. In fact, it became more and more apparent, as the new composite syllabus was evolving, that our efforts would be in vain unless we allowed sufficient time to elapse before proposing the adoption of the revised syllabus for teachers to prepare themselves for their new tasks. The syllabus was designed to do away with the compartmentalized teaching of arithmetic, algebra, geometry, trigonometry and some advanced topics . . . and to provide students of grades 7–12 the richest, most meaningful, and functional mathematical education they could be given at each grade level. No matter at what grade level a student discontinued his study of mathematics, he was assured of having all those phases of arithmetic, algebra, geometry, and trigonometry he was capable of appreciating, understanding, and using at that time.

To do this job effectively, a teacher could no longer claim to be a teacher of arithmetic, algebra, geometry, or trigonometry as it so frequently happens today. He must be a teacher of secondary school mathematics in its broadest sense. The content of each grade, while emphasizing one of the major areas of mathematics, includes those instructional materials from each of the other areas which are appropriate for purposes of correlation, increased understanding, and practicality.

The New Conception of Professional Preparation for Teachers of Mathematics

What has been done in New York State along the lines of mathematics curriculum revision does, I believe, indicate the trend in curriculum revisions throughout the country. The evolving curriculum of secondary school mathematics is basically sound and defensible. The major difficulty in putting the program into effect immediately lies in the fact that the traditional preparation of teachers of mathematics (and their experiences in teaching arithmetic, algebra, geometry or trigonometry separately) has to be supplemented by additional professional study. To teach mathematics effectively under this plan the teacher must know the subject matter of all the courses presently taught in the secondary schools, and also develop the teaching techniques for showing the interrelations between the various phases of mathematics, both theoretical and practical. This additional training is now being provided in many cities and smaller local units by "in-service" courses under the auspices of the State Departments of Education, State Universities, or local supervisor's of mathematics.

From the unsolicited testimonies of mathematics teachers (both experienced and cadet) it is quite evident that all is not well with our present graduate and undergraduate programs for preparing teachers of secondary school mathematics.

Let us examine the program usually prescribed for cadet teachers of mathematics —and see how well it meets the needs of our present day teachers.

When a young college student decides to prepare for teaching mathematics his professional program is pretty much prescribed for him by college regulations, or state regultions, or both. The professional courses usually prescribed for cadet teachers are primarily of two types; namely, 1)

General Courses in Education, such as Psychology, Principles of Education, Philosophy of Education, History of Educa tion, plus a course or two on General Methods and Techniques of Teaching. 2) Pure Mathematics courses on the college level, such as Higher Algebra, Higher Geometry, Advanced Trigonometry, Analytic Geometry, and the Calculus. In addition to these courses, the prospective teacher usually is given a two or three hour professional course, supposedly designed to prepare him to teach (and in many cases to learn) the mathematics of the secondary school level.

I invite each of you to reflect a moment on this situation. We cannot deny that this is the predominant pattern followed even to this day in most of our colleges and universities. You and I who are vitally interested in improving the professional preparation for teaching secondary school mathematics are not necessarily dissatisfied with the present course offerings in General Education on the one hand, and in Pure Mathematics on the other. The courses we have just enumerated really have a very important place in the total program. What we are saying in effect, is that our present inadequate preparation for doing an effective job of teaching junior and senior high school mathematics can be directly attributed to the fact that our present training program does not provide the necessary experiences in the wide range of subject-matter on the secondary school level, not sufficient time for developing the proper techniques for presenting the subject-matter clearly and interestingly.

I am reminded of some actual data I collected during the past eight years at a highly reputable Teacher's College in the east. Most of the students in this Teacher's College were selected on the bases of personality tests and high academic ratings on state examinations. It was not uncommon to find students whose academic achievements in high school subjects averaged 98% over a period of four years. Their knowledge of junior and senior high school mathematics was very satisfactory at the time they embarked on their Teacher Training Program. During the first three and one-half years of their college work, they have taken the usual courses in General Education and Collegiate Mathematics, including the Calculus, and in some instances, Theory of Equations, Differential Equations, Mathematical Statistics, and Advanced Synthetic Geometry. In the spring semester of their junior year, during the time they were enrolled in the methods class (a prerequisite to their practice teaching) they were given a composite test on the subject matter of secondary school mathematics. The results were very revealing, to put it mildly. These cadets who showed such outstanding promise at the beginning of their training period really knew less (after $3\frac{1}{2}$ years of study) about the subject they were supposedly preparing to teach than they had known before they ever entered the "Teacher Training Program."

It became quite apparent that something was radically wrong with the professional program for the education of cadet teachers. The traditional course offerings in General Education and Pure Collegiate Mathematics were not enough to prepare them adequately for the tasks ahead. You and I certainly would not think very highly of the medical profession, for example, if they turned out young doctors who knew less about medicine and medical science than they knew before they entered medical school.

What I have just said about this particular Teacher's College could be said about nearly every other teacher preparing institution in the country. For some reason or other the professional schools of education have not kept pace with the rapid changes which are taking place in the class rooms of our public schools. Too often do we find the University Professors of Mathematics unaware of the changes that are taking place in the content of our secondary school courses and, what is even worse, many are not even concerned

about the situation other than as it affects the students who enter their freshman classes in the University. On the other hand, those who have charge of the general courses in education in our professional schools are not expert enough, by their own admission, in the subject-matter areas to relate the general principles of education to the specific problems in all the major subject matter fields. This is not true of all college professors of Mathematics or professors of courses in General Education, but I do believe it to be quite true of institutions now permitted to train teachers of mathematics whose main purpose is other than that of preparing teachers for the public schools. Teacher's Colleges, in the main, are in a better position to provide the necessary experiences to produce first-class teachers.

If you and I look in retrospect to our college days, we see how wholly inadequate our professional preparation has been for meeting the situations we face today. Of course, we would not expect that the training program in those days be geared to the problems of a generation in the future, but neither should we expect that a professional program geared to the problems of a generation ago be adequate to cope with the problems of today.

I am happy to state that the University of Illinois through the Committee on the Preparation of Secondary School Teachers of Mathematics at this moment is studying ways and means by which to revise the teacher training program in mathematics so as to meet the needs of teachers in actual class room situations. They are seeking the advise of Illinois' teachers of mathematics, the university professors of mathematics, and the general educationists in building a dynamic and truly functional program.

It becomes apparent that before any positive action can be taken to improve the present preparation of mathematics teachers, we must come to some agreement as to what our professional "needs" really are and then to revamp our sequence of courses accordingly, or to build new ones as required.

I submit the following list of "professional needs" as they were revealed to me through sincere, personal requests for professional aid and guidance by class room teachers.

1. *Philosophy.* Somewhere in the professional program teachers should have ample time to discuss with experts in the field some of the major issues of mathematical education, so that they can develop a functional pattern of creative action. For instance, there has been an increasing amount of interest among high school mathematics teachers during the past few years, in teaching geometry to develop "clear thinking." There are many mathematics teachers who believe that the study of geometry does offer unusual opportunities for studying the principles of clear thinking. They believe that direct attention should be given to those prinples and to their application in various types of situations. Why has not more been done about the teaching of geometry for transfer of training to life situations? Where do we stand on this issue? If we thoroughly believe in it, should we not bend every effort to achieve our goals?

2. *A Rapid Review of Secondary School Mathematics.* Teachers feel the need for having a rapid review of the mathematics of the secondary school before they are asked to teach the subject. Many have suggested that this rapid review of high school mathematics *should be combined with appropriate techniques for presenting it.* This would demand readjustment of our total program in order to provide 2 or 3 courses of approximately 3 hours each. In this readjustment teachers have suggested that we include the preparation for teaching critical thinking in life situations, and a thorough treatment of approximate computation.

3. *The Use of Audio-Visual Aids and Mathematical Instruments in Vitalizing the Study of Mathematics.* Teachers are especially enthusiastic about the benefits to be

derived through the use of audio-visual aids to instruction. They want to know not only what films, film strips, models, slides, charts, mathematical instruments, and so on, are available for use, but also to learn how to vitalize their regular class room instruction through the proper use of these teaching aids. The Army and Navy have really shown how effective instruction becomes when approached through the use of audio-visual aids, and especially designed model instruments.

4. *The Need for Class Room Experimentation.* The teachers feel that there is a great need for class room experimentation to find the major difficulties in learning the various phases of mathematics. They feel that the training schools do not always provide the situations comparable to those which are to be found in the public school. Hence, teachers themselves should be given opportunities to carry on experimentation through the guidance and supervision of the university supervisors. These experiences might well be arranged through the medium of *field studies.* A well conceived and efficiently conducted program of experimentation, directly in the public schools, is seldom found today. Such a program would be eminently worthwhile in improving the conditions as they now exist.

5. *Special Courses.* Many teachers have suggested that a good course on statistics, including tests, measurements, and overall evaluation techniques be worked out cooperatively by the Professors who normally teach Mathematical Statistics, and those who teach General Educational Statistics, to build a composite course consisting of the mathematics of Statistics and the educational applications. Such a course would give them the background they need to teach the unit on Statistics now included in modern courses in junior high school mathematics and at the same time give them better preparation for reading and interpreting research literature and (or) applying statistical methods in their own experimentation.

6. *Modernizing Junior High School Mathematics.* Teachers feel that they could do a more effective job of teaching the general mathematics courses in the junior high school if they had a more adequate background in the social, business and industrial aspects of arithmetic, if they knew more about informal Geometry, techniques for teaching Elementary Algebra, and had richer experiences with practical applications of all sorts. They need not only a list of practical applications taken from industry, business, and the like but a better understanding of the situations out of which these problems grow. They feel that a teacher's course in General Mathematics, dealing both with subject-matter and methods of presentation, would be extremely valuable in remedying the deplorable conditions which now exist.

7. *The Adaptation of Instruction to Pupils of Varying Abilities.* There have been many attempts made to adapt instruction to the capacities, needs and interests of individual pupils. Some have tried ability groupings, more frequent promotions, differentiation of assignments and the like; these attempts at adaptation are greatly impeded by the lack of appropriate textbooks. It is the opinion of many teachers that the adaptation of instruction will only become effective under a new concept of textbook writing in which the author addresses the students directly and writes in language which is easily read and understood. The gifted child who uses such instructional materials can proceed at a rate commensurate with his ability. The slow learner is not made to feel out of place because of his slower rate of progress. Mastery of subject matter is made the criterion for passing the course. Under such a plan promotion can be made as frequently as the administrative organization of the school will allow. A program based on individualized instruction is possible, both administratively and instructionally.[2]

[2] Snader, Daniel W. "A Program for Indi-

8. *Teachers of Mathematics and the Professional Program.* Many young teachers, torn with conflicting emotions, have confided in me their feeling that collegiate mathematics for prospective teachers should be taught by professors who are genuinely interested in preparing teachers for the secondary schools. They also have the feeling that their education courses, whenever possible, should be taught by professors who are familiar with secondary school mathematics so as to be able to apply educational theory to the problems that arise in the teaching of mathematics. If this suggestion is sound, and possible of implementation, we should have a professional school on the campus of every university in which the subject matter courses and education courses would be presented by people who are professionally trained for this particular responsibility.

9. *Counseling Service for Teachers of Mathematics.* It would be of invaluable service to teachers in the field to have Teachers Colleges and University Colleges of Education provide a sort of "Counseling Service" or "Clearing House" which would be at their disposal for consultation on questions of professional interest. The problems referred to this Counseling Service could be answered either directly or through a regularly published bulletin distributed by the institution for the use of all mathematics teachers. The personnel required to assume the responsibility of this Counseling Service might well consist of one member from the university who is interested and actively engaged in the preparation of mathematics teachers; one member of the State Department of Education, probably the State Supervisor of Mathematics; and three public school teachers of mathematics, one from a large metropolitan school system, one from a small city school, and one from a rural consolidated high school; and in addition, a promising cadet teacher of mathematics enrolled in the university.

10. *In-Service Training of Teachers of Mathematics.* Teachers who have become acquainted with work-shop techniques have usually been enthusiastic in their endorsement of this method of professional study. They prefer it to the traditional courses in education and in mathematics which are so often offered for credit at so many dollars per point for a pre-determined number of class hours of attendance.

Summary: I have attempted in this discussion to review hastily the present conditions under which teachers are operating. I have attempted to show that the traditional preparation of teachers of mathematics, though perhaps adequate for teachers in the schools of a generation ago is not now fulfilling the professional needs. The changes in the curricular content of our secondary schools, especially that of the junior high school, are of such nature that it is almost impossible for teachers to prepare themselves adequately for their work under existing conditions. The study of traditional collegiate mathematics, though highly worthwhile in itself, does not adequately prepare our cadet teachers for the job ahead of them, unless these courses are supplemented by others which provide the experiences to which we have just referred. The same thing can be said about the General Education Courses.

I doubt whether there is any person intimately connected with the training of teachers who would disagree with me in this matter. All one has to do is to inquire of alert teachers in the field to find that this is a relatively accurate picture of the present conditions.

The teachers of mathematics have consistently asked us for help in developing a guiding philosophy; for a course including a rapid review of secondary school mathematics; for the opportunity to study the uses of audio-visual aids and mathematical instruments in class room work; to participate in experimental programs for improv-

vidualizing Instruction in Mathematics." THE MATHEMATICS TEACHER, March 1945.

ing classroom instruction; for special combined courses such as mathematical statistics and educational statistics; for a modern Teacher's Course in General Mathematics; for modern techniques in adapting instruction to the capacities and interests of individual pupils; for opportunities to study basic mathematics courses on the collegiate level under professors who are personally interested in preparing their students for teaching careers; for a Counseling Service for mathematics teachers through which problems of a professional nature can be channeled and workable solutions determined; for increased opportunities to participate in In-service Training Programs designed to meet the individual needs of class room teachers of mathematics.

These are the challenges which the teachers of mathematics lay at the feet of those who are in a position to act upon them. Each of you may have experienced one or more of these deficiencies at one time or another. Perhaps, there are some of you who would like to add one or more additional suggestions to this list. I personally would welcome the opportunity of speaking with you about them. The university is genuinely interested in providing you with every opportunity for professional improvement; make your wants known now, so that they may serve as a basis for constructive reorganization of the professional sequence of courses on both graduate and undergraduate levels.

Curriculum Trends in High School Mathematics

By E. R. Breslich
University of Chicago, Chicago, Ill.

The problem of selecting and organizing instructional materials for high school pupils is as old as the high schools. When these schools came into existence the courses in algebra and geometry then offered in the colleges were moved downward into the lower schools. Unfortunately these subjects had been organized by college instructors for college students and were in no sense planned to meet the needs and abilities of high school pupils. It was to be expected, therefore, that they would need to undergo considerable reconstruction. To the solution of this problem the mathematics teachers of the nineteenth century have devoted a great deal of time and effort.

Curriculum Trends

At the turn of the century the leaders in mathematics were still far from satisfied with the high school mathematics courses. Their efforts to bring about further improvement led to the introduction of a number of reforms which have influenced profoundly the teaching of high school mathematics to the present time. Perry of England sought the solution of the problem by recommending greater emphasis on the practical applications of mathematics, and on the use of experimental methods in the development of many of the general mathematical principles. Other leaders had come to the conclusion that not only mastery but also understanding should be major aims, and that they could be secured best by the frequent use of concrete materials and by methods of observation and experimentation rather than by logical deductions alone. Still others believed that much could be added to the interest and usefulness of the subject by bringing down into the lower courses some of the simple and valuable notions usually developed in the more advanced courses. Of far reaching importance was the movement of unifying the work in mathematics by means of such unifying factors as the function concept and functional thinking. These movements were not fads to be indulged in for a brief time, only to be dropped when interest in them ceased. Each has left its mark permanently on the content, organization and teaching of mathematics.

The courses in mathematics were further improved by the participation of the teachers of mathematics in such general educationalmovements as thedevelopment of tests and measurements, the junior high school, and the socialized curriculum. Thus from the beginning of the high school the story of the mathematical curriculum is one of continuous planning, progress and improvement—and hand in hand with these improvements went a steady increase in the popularity of the subject as reflected by the per cent of the pupil population attracted by it.

However, during the last four decades a noticeable change has taken place. Pupils have begun to avoid algebra and geometry in increasing numbers. Various explanations have been given to account for this change. Typical are the claims that pupils are experiencing greater difficulty with mathematics than with other school subjects; that for a large portion of the student body the existing courses go too far beyond their every day needs, even beyond their vocational needs, while not sufficient attention is given to the type of mathematics which most people need in the activities of every day life.

The falling off of the popularity of high school mathematics has been noted by the teachers with a great deal of concern. For, as modern civilization has become more and more complex, it also has become more mathematical than ever before. In-

deed, the uses of mathematics are still on the increase in numerous occupations, and the demand for men and women who had at least the mathematics taught in the high school is steadily growing. Moreover, many industries are now entering new fields of production and to make possible the development in these fields much research has to be done. In this they need men and women who are college trained and know college mathematics. Yet there is an increasing lack of persons qualified to take advantage of the great opportunities now offered to them. The existing shortage cannot be relieved for years and it will be felt especially in the years just ahead. For example, it has been estimated that the need for engineers[1] alone will increase by 1950 from the present 261,000 to 337,000. They will need all the mathematics they can absorb in the high school.

Furthermore, because people today can use advantageously a great deal of mathematics in the ordinary affairs of life it would seem that every pupil should study some mathematics in the high school irrespective of the choice of his future occupation. The interest of many will be served best by the traditional mathematical sequence, but for the others different plans have to be developed.

The realization that a single plan cannot satisfy the needs of all pupils had already come to the National Committee[2] of 1923. Instead of yielding to the popular demand "for a single standardized syllabus, by years and half-years, with a fixed order of topics and time allotment for each" the Committee found it advisable to present five plans, to allow the teachers sufficient freedom in developing "courses to fit individual needs and conditions." Furthermore the Committee warned against the mistake of regarding any one of the five plans superior to the others. However, there was considerable difference. While according to *Plan A* algebra is introduced in the eighth grade and continued in the ninth, and some trigonometry and demonstrative geometry is provided by the end of the ninth, the work of the same grade in *Plan E* is limited to simple algebra, informal geometry and numerical trigonometry, and includes no demonstrative geometry. The purpose of offering different plans was to make it possible "to give the pupils the most valuable information and training that they are capable of receiving in these years."

The need for more than one curriculum plan was again recognized by the *Joint Commission*[3] of 1940. The report presents two detailed outlines. The first recommends the study of algebra in the ninth grade "for pupils of normal ability who have had good training," and suggests a beginning of demonstrative geometry for classes of superior ability. Trigonometry is to be taught as a way of finding heights, distances and angles by use of the natural functions. After the ninth grade the plan adheres to the traditional sequence: demonstrative plane geometry for the tenth grade, algebra and trigonometry for the eleventh, and solid geometry and analytic geometry for the twelfth. The plan is "intended especially for pupils who expect to follow a profession that requires considerable mathematical training, or who prefer such a course."

The second plan proposes for the ninth grade a composite course consisting of arithmetic, graphic representation, algebra, numerical trigonometry, social mathematics, informal geometry and logarithmic computation. For those pupils who will continue the study of mathematics after the ninth grade the second plan, like the first, outlines work for the remaining grades. The Commission believes that no

[1] James R. Killian, "Tomorrow's Scientists," *The Lamp*, XXVIII (August, 1946), 23.

[2] Report of the National Committee on Mathematical Requirements. *The Reorganization of Mathematics in Secondary Education.* The Mathematical Association of America, 1923.

[3] *The Place of Mathematics in General Education*, published as the Fifteenth Yearbook of the National Council of Teachers of Mathematics. Teacher's College. New York City, 1940.

single type of curriculum can be devised which will meet the needs and abilities of all pupils, and that in some schools the sequential courses will be best for particular pupils while the interest of the larger group will be served best by the second plan. By providing more than one plan the Commission feels that some study of mathematics can be made a part of every pupil's education irrespective of whether or not he plans to go to college.

The thesis that the schools should "provide adequate training in mathematics for *all* students" has been further endorsed by the Commission on Post-War Plans.[4] "There are neglected groups of students"—says their report—"very large ones, whose needs cannot possibly be met in traditional courses." The Commission recommends that "the traditional courses should be reserved for "pupils who can do these courses." Three series of courses should be provided: sequential mathematics, related mathematics, and social mathematics.

A year later, a second report[5] of the Commission assigns to the high schools a dual responsibility in regard to mathematics:

1. *to provide sound mathematical training for our future leaders of science, mathematics and other learned fields.*

2. *to insure mathematical competence for the ordinary affairs of life . . . as a part of a general education appropriate for the major fraction of the high school population.*

Accordingly, the Commission proposes a "double track" plan in mathematics. Track I should be the traditional sequence. It is to be reserved for those pupils who have the requisite ability, desire, or need for such work. For the ninth grade it should offer algebra. The rest of the pupils will follow Track II. It will be general mathematics for grade nine and comprises the basic concepts of mathematics. Its function is to improve the necessary skills. The work for grades 10 to 12 is to continue in parallel courses.

Curriculum Proposals

The three foregoing reports disclose an unmistakable trend away from a single mathematical curriculum for all high school pupils toward one consisting of two or more plans, each designed to meet the needs of a particular group. They agree that one of these plans should retain the traditional sequential courses because they offer a strong mathematical training program, which to many men and women is often more important than any other school work they may take. From this group come the future engineers, professional scientists, research workers, and many others who will use much mathematics in their work. To the same group belong also most of those pupils who after high school graduation expect to continue their education in college. They will be benefited greatly by the training offered in these mathematical courses. Moreover, many pupils will choose the sequential courses for no other reason than that they are mathematically minded and find the subject enjoyable.

The content and organization of the sequential courses are by no means to be thought of as having attained a form not capable of further development. As needs become apparent they will be changed and improved. Right now there are some pressing changes which should not be deferred any longer. Recent studies have shown that high school graduates are not proficient in arithmetic. This criticism should not apply to pupils who have taken the mathematical sequence. If it does, some carefully planned instruction in arithmetic should be devised. Mathematics also should make a larger contribution to the problem of training for citizenship. Only too often the social applications

[4] First Report of the Commission on Post-War Plans, The Mathematics Teacher, XXXVII (May, 1944), 226–232.

[5] Second Report of the Commission on Post-War Plans, The Mathematics Teacher, XXXVIII (May, 1945), 195–221.

merely serve to motivate the work in mathematics, to make it more interesting and thus to secure better results. Social applications should be taught also for the informational value they possess. They should train the pupil to deal more effectively with the quantitative problems of the community and the nation. Criticisms of mathematics if carefully analyzed by the teachers can lead to improvement and can thus render important services to the subject.

The size of the group taking the sequential courses will vary with different schools. For some it will be the largest group. For others the situation will be reversed. For still others two groups may be equally large. Evidence seems to show that for the country as a whole it is the smaller group that will take the sequential courses.

The fact that the other group is not nearly as homogeneous as the first makes the problem of planning a suitable curriculum quite difficult. Furthermore, it complicates the teaching situation. Unless these classes are taught by the best teachers the results will be questionable. That the National Committee of 1923 was fully aware of the complexity of the problem is seen from the statement in that report that the work of the ninth year must be so planned as to give "the most valuable mathematical information and training which the pupils are capable of receiving in this year with little reference to future courses which they may or may not take."

A partial answer to the problem of determining this "most valuable mathematical information" may be obtained from a study of the proposals of the foregoing three national committees. To these may be added the recommendations of two other committees.[6] To be sure, these two reports were written to aid the armed-forces. However, the Commission on Post-War Plans rightly says that "it will be sensible to assume that the minimum Army needs with only slight modifications should be part of the education of all our citizens."

[6] Report of the Committee on Preinduction Courses in Mathematics, THE MATHEMATICS TEACHER, XXXVI (March, 1943), 114–124. See also the Report of the Committee on Essential Mathematics for Minimum Army and Navy Needs, THE MATHEMATICS TEACHER, XXXVI (October, 1943), 243–282.

BASIC MATHEMATICS

Although mathematical needs vary with individuals, schools, and school systems there exists a body of mathematical concepts, principles, and skills important in all vocations, necessary for intelligent citizenship, useful to all persons, and helpful in solving the quantitative problems encountered in everyday life. By selecting the proposals on which four or all of the five reports agree such a body of basic mathematics may be identified. This has been done and the results are listed below.

For convenience the materials have been grouped under the headings: arithmetic, geometry, algebra, statistics, graphical representation and numerical trigonometry which, of course, is not the order of arrangement for teaching purposes. These materials constitute a minimum of mathematics which might be made the starting point by those building mathematical curricula for groups of pupils not taking the sequential courses.

1. ARITHMETIC. It is desirable that arithmetic deficiencies of individuals and groups be disclosed at the beginning of a course by making use of diagnostic tests. With the information so obtained the teacher can plan remedial or additional instruction. It has been found to be a mistake to teach arithmetic in the high school as a topic by itself. Rather it should be related to other topics, to problems, and to practical applications. For example, in algebra and geometry there is an abundance of formulas in which arithmetic will be used in substitution and evaluation. In geometry arithmetic is closely related to measurement. In general, the place to

teach or reteach arithmetic is wherever weaknesss appears. Often it will be desirable to let the teaching be followed by practice and formal drill to make responses automatic.

a. *Basic concepts.* Clear understandings should be attained of the meanings of such basic concepts as a whole number, common fraction, decimal fraction, per cent, and ratio.

b. *The fundamental operations.* In helping the pupil become proficient in the fundamental operations it should be remembered that there is more to the arithmetical processes than rules to be used to obtain answers. In the high school emphasis must be on understanding of the arithmetical principles underlying the processes. The pupil must have clear meanings of these principles if he is to be able to use them intelligently.

c. *Ratio and proportion.* The concepts *ratio* and *proportion* should grow out of applications which occur in everyday life and which fall within the experiences of the pupils. Ratio is a very difficult concept to grasp. It is used in geometry when comparisons of statistical data or of any like measures are to be made. In the study of similar figures the ratios of corresponding sides lead to proportions. In numerical trigonometry the ratio concept again enters with the introduction of the sine, cosine, and tangent. Ratios are expressed in different forms: as common fractions, decimal fractions, and per cents.

d. *Square root.* The process of extracting square roots is frequently listed among the "obsolete" topics of mathematics recommended for elimination from the curriculum. On the other hand, the teachers of several high school subjects insist that it is sufficiently important to be retained. The fact is that square roots enter frequently in the solutions of problems arising in algebra, geometry, and shop and science courses. Hence, the pupil should know how to find them. Three methods are available. He may use a "table of square roots." These tables are satisfactory for small numbers. For numbers larger than 100 he needs to know some process of extracting the square root. One of these is the process by "division." It is based on the definition that a square root is one of the two equal factors of a number. One argument in favor of this method is that it offers good training in arithmetic. The pupil who has become acquainted with the formula for finding the square of a binomial is ready for the algebraic method which is based on this formula.

e. *Measurement.* Measurement enters frequently in the activities of everyday life. It is therefore necessary that the pupil be familiar with the common units of length, weight, area, and volume. The metric units should not be overlooked. He should acquire facility in the use of such instruments as ruler, tape, protractor, compasses, calipers, squared paper, t-square, and the right triangles with 45-degree and 30-to 60-degree acute angles. There is no scarcity of problems in measurement which may be taken from the sciences and other school subjects. The pupil should be led to recognize that all measurement is approximate and to understand the meaning of accuracy of performance, figure accuracy, precision and rounding off numbers.

f. *Percentage.* Percentage is not to be taught by the "case method." However, the ability to identify cases should grow out of an abundance of problems in which percentage is used, such as the problems dealing with profits and losses, installment buying, borrowing money, saving deposits, and discounts. The first two cases of percentage are to be solved by arithmetic. Later all three cases are solved by means of the percentage formula.

g. *Tables.* Tabular representation is a popular arithmetic method of recording statistical number facts. The reader is expected to be able to interpret the tables so as to derive worth while information from them. Mathematics has an opportunity to provide ample training of this type in the study of statistical tables, tables

of squares and roots, and the three- and four-place tables of the trigonometric ratios.

h. *Verbal problems.* Much of the training in arithmetic computation is to be obtained by solving problems. Hence, pupils must acquire power in problem solving. By being taught an effective technique they will learn how to attack problems intelligently, to identify the given facts and those to be found, to recognize relationships between them, to select the right processes to be used, and to check the answers. In order that the training in arithmetic to be derived from the problems be as large as possible it is necessary that the situations in the problems be familiar and varied so that they are understood by the pupils. The following are suitable types of problems.[7]

1) Problems of the home, dealing with accounts, budgets, expenditures, bookkeeping, shelter, food, clothing and medical care.

2) Problems of the community, such as taxes and property insurance.

3) Problems of banking, as for example those related to savings accounts, checking accounts, mortgages, and borrowing money.

4) Problems of investments, particularly postal savings, social security, stocks and bonds.

5) Problems of communication, concerned with travel, transportation, parcel post, and sending money.

2. Geometry. Geometric facts, principles and relationships enter into every pupil's experiences in his school work and in his out of school life. Without some instruction he may be unable to understand them. Indeed, he may be unaware of their existence. The following outline lists the phases of geometry with which every educated person should be familiar.

[7] A helpful discussion will be found in *The Role of Mathematics in Consumer Education.* The Commission on Post-War Plans, 1201 Sixteenth Street, N.W., Washington, D. C., 1945.

a. *Geometric concepts.* The pupil should learn the meanings of such concepts as point, lines (straight, broken, curved, parallel, perpendicular), angles (acute, right, straight), triangles (isosceles, equilateral, right), quadrilaterals (square, rectangle, parallelogram, trapezoid), polygons, and circles. Furthermore, he should recognize, understand and be able to make sketches of cubes, prisms, pyramids, cylinders, cones and spheres. He should know what is meant by length, area, volume, congruence and similarity.

b. *Direct measurement.* In problems involving measurement arithmetic and geometry are closely related. Hence, the details about direct measurement have been stated above in the outline for arithmetic.

c. *Properties of geometric figures.* The pupil must become familiar with the important properties of the common geometric figures if he is to make use of his knowledge of geometry. To illustrate, he should know the relationships between angles formed by intersecting lines, the angles of a triangle and the sides of the right triangle, and the conditions which make two triangles congruent or similar.

d. *Indirect measurement.* When the pupil finds that he cannot determine unknown lengths by applying the unit of measure directly he is ready for the study of indirect measurement. He learns to obtain measures by use of squared paper, scale drawings, formulas for area and volume, congruence and similarity of figures, and trigonometric ratios. Incidentally he is to be trained thoroughly in accuracy in measuring and computing and in preestimating lengths, heights, areas, and volumes.

e. *Geometric constructions.* The purposes of the geometric constructions are to develop skill in the use of straight edge and compasses and to be able to construct good geometric figures such as isosceles and equilateral triangles, squares, parallelograms and tangents to a circle. The pupil should master the basic constructions of drawing perpendicular lines, of dividing a

segment into two or more equal parts, of bisecting an angle, and of drawing an angle equal to a given angle.

f. *Cultural values of geometry.* The outline above has stressed the informational and practical values of geometry. However, the cultural values also deserve a prominent place. The pupil should be led to appreciate the geometric designs and ornaments on buildings, furniture, floor coverings, and numerous other objects. Geometry should give him a better understanding of the solar system and the universe. He will be impressed with the value of mathematics when he reads about effort of the race to refine ways of measuring and to develop standard units of measure, and when he learns about the great contribution of geometry to the progress of civilization.

3. Algebra. Algebra is becoming increasingly important. The algebraic method simplifies the solutions of many problems which are too difficult when only arithmetic is known. Quantitative statements which confuse the reader can often be made clear by using algebraic notation, formulas and equations. In school work algebra is a valuable aid in the study of various school subjects. It is indispensable in numerous vocations. A limited amount of algebra is therefore worthy of a place on the curriculum planned for pupils who are not in the sequential courses. The following outline indicates the type of algebra to be taught.

a. *Algebraic concepts.* Symbolic notation is to be taught whenever need arises and when it clarifies relationships or facilitates correct thinking. The following concepts are essential: literal number, signed number, exponent, radical, formula and equation.

b. *Fundamental operations.* Emphasis in teaching should be on understanding, and meaningless manipulation is to be avoided. The operations to be taught are addition, subtraction, multiplication, division and some factoring. Simplicity of the work should be secured by using mostly simple examples of monomials, polynomials, signed numbers, exponents, radicals and fractions. Thus in factoring only the first three cases should occur and there should be no complex binomials, trinomials and polynomials. The teacher should remember that the laws of algebra can be understood best in simple exercises.

c. *Equations.* Equations are used to solve problems, especially those not readily solved by arithmetic. They should be simple types in one or two unknowns. Some easy quadratics in one unknown may be introduced. For the purpose of clear understanding the steps leading to the root of the equation should be carefully rationalized. They should be based on the processes of addition, subtraction, multiplication, division and finding square roots. Such purely mechanical processes as transposition should be avoided.

d. *Ratio and proportion. Variation.* His experiences with equations will enable the pupil to improve his skill in solving problems which lead to proportions. Simple problems in direct variation may also be solved by the use of proportion.

e. *Formulas.* Various aspects of the formula should be taught.

1) A formula is a shorthand rule, as the rule for finding the distance traveled at a uniform rate in a given time. Likewise formulas are used to compute interest, areas and volumes. Many formulas should be derived by the pupils.

2) A formula is an algebraic way of expressing relationships, such as dependence of one variable on one or more others. The Centigrade-Fahrenheit relationship is a typical example.

3) A formula may be a general solution of a given type of problem.

4) A formula is an equation. Both are solved by the same methods.

5) A formula may be represented geometrically by a graph.

6) *Problem solving.* Algebra like arithmetic and geometry aims to increase the pupil's power to solve problems. Some of the steps in the technique of solving prob-

lems are alike for algebra and arithmetic, but in algebra the pupil derives and solves an equation, while in arithmetic he chooses and employs the fundamental processes. In both cases he checks the solution by substituting it in the problem.

4. STATISTICS. The pupil encounters statistical data in his reading of newspapers and magazines, in his school studies, in advertisements, in fact most anywhere. Hence he should be taught as much of the vocabulary of statistics as he may need. To understand what the statistical terms mean he must find arithmetical means, medians, and modes of a number of given measures, particularly of the data presented in simple statistical tables. Familiarity with the use of index numbers is also desirable. Pictograms, statistical graphs and graphs of frequency distributions should be studied to clarify the meaning of statistical data. The work in statistics should be closely correlated with arithmetic and algebra.

5. GRAPHICAL REPRESENTATION. Instruction in graphical techniques is important because they are widely used to clarify statistical facts, to make comparisons of data, and to disclose trends. The foregoing outline has made it clear how closely graphs are related to arithmetic, algebra and geometry. They should therefore be taught in connection with these subjects. The following abilities should be developed.

a. To read, understand, and interpret graphs.

b. To make comparisons between data represented by bar graphs.

c. To recognize trends reflected in line graphs, and to note central tendencies.

d. To construct simple statistical graphs mainly for the purpose of attaining a better understanding of graphical techniques.

e. To make graphs of simple formulas.

f. To show the use of graphs in solving problems.

6. NUMERICAL TRIGONOMETRY. The problem of determining indirectly distances, heights and angles has been encountered repeatedly in this outline. It came up in learning about scale drawings, algebraic proportions, congruent and similar figures, and the Theorm of Pythagoras. Gradually the pupil has realized the limitations of those methods. They are tedious and not very accurate and he will appreciate the simpler and more accurate method of trigonometry which is used in the practical work of the surveyor. There is ample problem material in his surroundings. He is interested in finding heights of flag poles, smoke stacks, and buildings, and the use of steel tape and transit of the surveyor will be a real attraction. Thus, the basic facts of trigonometry are taught and learned through use. To deepen his knowledge of the sine, cosine, and tangent ratios he should construct small trigonometric tables including angles of 10°, 15°, 30°, 45° and 60°. This is followed by practice with three- and four-place tables. The major topic will be the solution of right triangles by use of trigonometric ratios and tables. As previously in problems of measurement the question of accuracy and precision is given careful attention.

OUTLOOK FOR MATHEMATICS

Since the uses of mathematics are on the increase in industry, manufacture and business, in numerous occupations in which young people expect to engage, in other school subjects, and in the affairs of every life, the work in mathematics begun in the elementary schools should not be discontinued in the high schools. They should provide some further mathematical training for all pupils. Recently there has been evidence of a renewed interest in mathematics by parents and administrators of the schools. The teachers of mathematics should take advantage of this interest.

Many of those who criticize the mathematics offered in the high school do so because they are really interested in the subject. They are against any general mathematical requirements of the courses in the

traditional sequence. They have no objection to them when they are taught by good teachers to a select group of pupils who will need that type of mathematics in their future professions or vocations. For the others they demand a different selection and organization of subject matter.

It has been shown that during the last twenty-five years the trend of the proposals recommended for high school mathematics has been exactly in that direction, and the foregoing outline comprises a minimum of concepts, principles and skills. This material is not only worth while to all pupils not taking the sequential courses, but it is basic to all study of mathematics, even to the sequential courses. This minimum should be thoroughly mastered. The amount and kind of subject matter to be added to the minimum to round out a course will vary for different types of schools and pupils. Much of it is available in text books on junior high school mathematics and ninth-grade mathematics.

So far, in this paper, attention has been given mainly to the recommendations relating to the instructional materials to be included in the courses planned for pupils not in the traditional sequence. There in no intention, however, to disregard the many excellent suggestions as to ways of organizing and presenting these materials and as to the objectives to be attained.

All reports emphasize practical applications and problems relating to life situations. The majority of these problems should be real and impress the pupil as worth while. They supply the motive for the study of the mathematical processes needed to solve them. Since the mathematics involved in the solution of problems is usually of a simple type, the processes and principles to be learned will be exhibited in simple mathematical situations, and understanding is thereby facilitated. Moreover, experiences with problems will develop power to deal with the quantitative situations which the pupils will meet as adults in everyday life. Finally, from the social situations involved in the problems the pupils may gather much valuable information.

While the practical values should have an important place in mathematics, the disciplinary and cultural values should not be overlooked. Abstract mathematics does not hold very long the interest of many pupils. Others tire of the practical applications. But glimpses of the relation of mathematics to the history of the human race appeal to most pupils. The stories of the development of number, of the practical geometry invented by the Egyptians to carry on extensive projects in irrigation and to build their temples and pyramids, and of the algebraic symbolism of literal numbers, signed numbers and exponents are all intensely interesting and will add to the attractiveness of the course.

One of the most effective motives for the study of mathematics lies in the needs of other school subjects. Teachers of various school subjects often complain that the pupils do not know how to use their mathematics. This offers an opportunity for teacher conferences in which the specific needs of these subjects can be identified. They can then be provided for in the mathematics course.

Fear has been expressed by some teachers that the double track plan will fail because pupils have come to regard any course other than the traditional sequence as inferior. Responsible for this reputation is the practice of crowding into shop mathematics, commercial arithmetic, consumers mathematics and similar courses all pupils of inferior ability, non-college pupils, and those who failed in algebra. Many pupils have felt humiliated when placed into such a course. It should be made very clear to parents and pupils that the difference between the courses of any double track plan lies mainly in the aims set up.

In the general mathematics courses emphasis will be on understandings of rules and processes, on training in study habits,

on neatness, orderliness, concentration and perseverance. In them will be found college and non-college pupils. Individual differences of ability there will be, but these problems will be taken care of by the same methods that are used in any course whenever individual differences appear.

An important question in the administration of the double track plan is that of time allotment. Several ways are open. The general mathematics course may be planned for one year, preferably the ninth grade, for five periods a week. This avoids some serious administrative problems. The objection is that after the ninth grade a large per cent of the pupils will drop out and before they are graduated much of what has been taught will be forgotten because of disuse. The difficulty can be overcome by allotting three days a week for the ninth grade, and two days for the tenth. Some schools would rather assign one period a week extending over the entire high school period. Obviously there are administrative difficulties in this arrangement. However, an advantage is the fact that pupils will be doing some work in mathematics a large part of the time which they spend in the high school. Also it will be easier to adapt the course to the pupils' mental growth due to wider experiences. Many applications, such as taxation, insurance, and investments now usually taught in grades 8 and 9 could then be deferred to the time when the pupils are old enough to appreciate and understand them.

The aims of the general course will be difficult to attain, and classes should therefore be taught by good teachers with broad experience, who understand high school pupils, and who can make mathematics sufficiently interesting to inspire them.

It is now over two years since the last of the reports of the national committees was made. They have been read by the teachers. They have been widely discussed in teachers conventions. The question is: are teachers and administrators taking the reports seriously, and are the recommendations being put into practice on anything like a nation-wide scale? With the present interest it should not be difficult to make a course in general mathematics popular and to attract pupils in great numbers. The prospect for the future of high school mathematics should be extremely favorable.

Association of Mathematics Teachers of New Jersey State Teachers College, Newark, New Jersey Saturday, March 6, 1948

PROGRAM

STUDIES IN APPLIED MATHEMATICS

10:30 Aptitude Testing, Employee Selection and Placement.
Dr. R. B. Selover, Personnel Research Director, Prudential Insurance Company, Newark, New Jersey.

11:00 The Use of Statistical Analysis in Controlling Quality.
E. H. MacNiece, Director of Quality Control, Johnson and Johnson Company, New Brunswick, New Jersey.

11:30 Some Recent Trends in Applied Mathematics.
Dr. John H. Curtiss, Chief of the National Applied Mathematics, Laboratory Division of the National Bureau of Standards, Washington, District of Columbia.

12:00 Discussion.

12:30 Showing of Films for the Teaching of Mathematics.
1. *Precisely So*, Chevrolet Motors, Division of General Motors Corporation, Detroit, Michigan.
2. *Behind the Shop Drawing*, Jam Handy Organization, 2821 East Grand Boulevard, Detroit 11, Michigan.

A Usable Philosophy in Teaching Arithmetic[1]

By DENNIS H. COOKE
The Woman's College of the University of North Carolina, Greensboro, N. C.

AS FAR BACK as I can remember arithmetic, number, and applied mathematics have been interesting to me. The teaching of number and arithmetic has been a hobby of mine for twenty-five years. At the ripe old age of eighteen, in taking inventory of my ambitions and my means of attaining them, I found on the credit side of the ledger that I had attended college for one year, had passed all my work creditably, and had a burning passion to finish my college degree. On the debit side of the ledger I found that I had spent all of my money and would need to earn the necessary funds with which to finish the three remaining years of college for the completion of the degree. My father being a close friend of the chairman of the board of education in my home town, it was arranged that I would teach the sixth grade in the local elementary school.

Having had one year of general freshman college work consisting of Latin, chemistry, English, history, and mathematics, of course, I had had no psychology or pedagogy or anything else that would help me teach this sixth grade; so, in desperation, I attended a county teachers' institute for five or six weeks during the summer. There I learned about teaching writing, spelling, geography, and arithmetic, but I devoted practically all my time to the course in teaching arithmetic. So at the appointed time in September, the appointed place in my home town where I had been a student only a year previously, with one year of college preparation, I reported to my assignment to teach the sixth grade. Aside from teaching a younger brother in this grade and the fact that the principal never entered my room during the entire year, I was off to a fairly good start. But in spite of my interest in teaching arithmetic I was not getting results. Since it was the season of the year when the World Series Baseball games were being played, I hit upon the idea of organizing my class into the two baseball teams that were playing the series for the purpose of stimulating interest in the arithmetic work. The results were really amazing. Throughout the years, beginning with this early experience twenty-five years ago, I have been trying to find better ways of teaching arithmetic; so here I am to share with you some of my experiences and ideas concerning "A Usable Philosophy in Teaching Arithmetic."

Webster defines "philosophy" as a "systematic body of general conceptions or principles, ordinarily with implication of their practical application." According to this definition of a philosophy I am not required to set forth a scientific proof for my usable philosophy in teaching arithmetic, and none will be attempted; suffice it to say that I believe the philosophy that I shall propound to be sound and usable. Knowledge for its own sake is of very little value; in fact, knowledge must be usable and applicable to warrant our efforts in acquiring it. Of course, I believe that my philosophy is usable. But let me hasten to my topic more specifically.

My first point has to do with the placement of topics in arithmetic. Of course, a topic should not be studied in arithmetic until the child is psychologically prepared and ready for it. Granted that the child is psychologically ready for a given topic, still he should not study this topic until he has some social utility or need for it. For example, psychologically and from the point of view of readiness, some children can learn effectively all the addition, sub-

[1] Address before the National Council of Teachers of Mathematics, Atlantic City, New Jersey, February 28, 1947.

traction, multiplication, and division facts before they complete the first grade, but we are not warranted in teaching all of these facts to first graders, because many of these facts have no social utility for first graders. There comes a time, however, when the child is not only psychologically ready for all of these facts, but a time when he needs them in his play and in his number relationships with other children as well as with adults. Those topics that are introduced to the child only when he is psychologically ready for them and only when he has social utility for them will most likely be understood by him, and for him these topics will have meaning.

Possibly through the third grade none of the work in number, as well as in many of the other subject-matter areas, should go beyond the child's immediate experiences and activities. But beginning about the fourth grade, the child should begin to store away some of his fundamental learnings for later life, yes, even some of those in arithmetic. As the educational level increases, a larger and larger percentage of his fundamental learnings should be of the store-away type. For example, much of the work in percentage for the sixth grader has little immediate use, but the student could not do the work of the seventh and eighth grades and high-school mathematics without all the work in percentage that he learns in the sixth grade. The student who drops out of school at the end of the sixth, seventh, or eighth grade learns much about percentage in these grades that he can apply and use only in adult life. But if he did not learn these things in these grades, where would he learn them? Let me hasten to add that in the field of number more socially useless material has been taught than in probably any other elementary school subject-matter field. Fortunately, the tendency is toward an elimination of much of it and a stepping up of much of the balance in the matter of grade placement.

In the second place a usable philosophy of teaching arithmetic dictates that practice and drill not be wasted on things that the students already know and can do beyond the point of maintaining that which is known. Obviously the amount of practice and drill needed on any topic or process in arithmetic is dependent largely on the difficulty of the topic or process. For example, the meaning and use of percentage will require much more time and drill to master than will the process for checking subtraction. The multiplication facts with 9 are much more difficult to master and more practice is needed on them than the multiplication facts with 2. Division with two figures in the answer and with a remainder is more difficult and requires more time than simple division with one figure in the answer and no remainder. Multiplication with carrying is more difficult and requires more practice than multiplication without carrying.

But what is more serious is the assumption on the part of many teachers that the same amount of time and drill are required by all students to master a given process, number fact, or concept. To describe a teacher who taught with me twenty-five years ago illustrates this point. As principal of a departmentalized elementary school, I had a teacher of arithmetic who was a drill master, and a very proficient one at that. She prided herself on the fact that her students seldom missed an addition, subtraction, multiplication, or division fact. To achieve this end many of the recitations of her arithmetic classes consisted solely of oral drill on these facts. She had very little written work with the facts as such. In this departmentalized elementary school the teachers changed classrooms rather than students. As this teacher left a drill session in one room to go to another room for the same purpose, as she went down the corridor and as long as she was in hearing distance of the class she was leaving you would hear something like "5 8's" from the teacher in the corridor, and from within the room you would hear "40" in unison from all the students. This procedure continued as long as she could hear

the students and could be heard by them. When in hearing distance of the next class to which she was going, this teacher would begin something like "9 and 8" and from within the room she was approaching would come the answer by all the students in unison "17."

She proceeded from class to class during the drill seasons in this manner. Her students learned the number facts and learned them well although at a terrific price. Many of the students already knew all of the facts well enough to leave them except for a small amount of drill for maintenance purposes. But every student received the same amount of drill on the facts regardless of the extent to which he knew them. Not only this, but each and every fact received the same amount of drill. If today's assignment were the addition facts, for example, the teacher started in with the 2 facts, in random order, and proceeded through all the facts until each and every fact had been called for once. Obviously, too much time was spent on the easy facts and too little on the difficult ones.

If this teacher had followed an adequate teach-test-reteach program, she would have avoided this gross waste of time; so the third phase of my usable philosophy of teaching arithmetic is the recognition of and provision for individual differences among the students through diagnostic and remedial procedures. At the end of each unit of work the teacher should administer to every student appropriate diagnostic tests from which it can be determined on which topics individual students need further practice. Without such diagnosis students frequently go over and over again arithmetical facts, processes, and concepts which they know sufficiently well except for maintenance purposes. Although practice and drill are indispensable in effective teaching, they should be had when they are needed. Time should not be wasted on practice and drill where they are not needed.

While a large amount of diagnostic work and a limited amount of remedial work in arithmetic can be done with groups of students and on a group basis, the most effective diagnostic and remedial work is done with individual students and on an individual basis. This statement has been proven by a number of researches among which are two of my own reported in the *Peabody Journal of Education* in 1931 and 1932.[2] It is of little value, however, for a teacher to engage in elaborate diagnostic procedures unless the diagnoses are followed up by proper and adequate remedial procedures. The *remedial* exercise must be adapted to the needs of each individual. Groups of students with the same difficulties can and should be organized for group instruction. In the remedial procedures every effort should be made to eliminate faulty habits, such as counting, careless work, and so on. Faulty and inefficient methods of work should, of course, be corrected. The teacher and student should cooperatively set up standards of attainment that the student can hope to achieve. Then higher standards should be set upon achieving the lower ones. An indispensable remedial technique of considerable value is the teaching of methods of checking all work. Students should be taught how they can select the proper remedial exercise for their difficulties. Good arithmetic teaching does not expect or demand the same rate of progress for all students. Finally, in the remedial procedures, students should be taught good techniques of problem solving, which I shall discuss next.

The fourth phase of my usable philosophy in teaching arithmetic is a logical approach to problem solving. I shall discuss this point in terms of one-, two-, and three-step word problems. Very simple one-step word problems can be presented to the student as soon as he has developed a working vocabulary and a few number

[2] Dennis H. Cooke, "Diagnostic and Remedial Treatment in Arithmetic, I and II, *Peabody Journal of Education,* November, 1931, pp. 143–151 and November, 1932, pp. 168–171.

concepts, in some instances as low as the second or even late in the first grade. Beginning third graders can usually handle one-step word problems of the type: *There are 5 trees in this yard and 3 in the next yard. There are how many trees in both?*

There should be a series of logical anticipations as forerunners to two-step word problems. The first anticipation on the part of the student of a two-step problem should be made near the middle of the third grade where some of the problems ask two questions, such as, *Jane's lunch cost her 21¢ each school day last week. How many school days are there in a week? How much did she spend for her lunches for the week?*

The second anticipation of a two-step problem should come near the end of the third grade where there are problems using the answer to previous problems. For example, problem 1 could read, *Jimmie bought 2 boxes of candy mints. There were 5 pieces in each box. How many pieces did he have?* Then problem 2 which is based on problem 1 would read, *How many pieces would that be for each of Jane and Jimmie?* (See problem 1.) This type of problem should be followed in the fourth grade by problems with two questions where the answer to the second depends on the answer to the first. For example, *Mother filled 6 boxes with cookies and candy. She had 65 cookies. She put 9 cookies in each box. How many cookies did she use? How many were left?* This is the third anticipation of a two-step problem. The fourth anticipation of a two-step problem should also occur later in the fourth grade with problems that have only one question but require two answers. For example, *How many dimes and nickels are there in 15¢?* Following these four anticipations of a two-step word problem, the students are ready for two-step problems near the end of the fourth grade, such as *Grandfather and Jimmie sold beans for $5.40 and carrots for 90¢. How much more than $6.00 did they get for both?*

Problems with three questions should be presented in the fifth grade as preliminary to problems with three steps. For example, *In the running broad jump John jumped $10\frac{1}{8}$ feet, Alfred jumped $11\frac{1}{2}$ feet, and Frank won with $12\frac{3}{4}$ feet. By how much did Frank beat John? By how much did Frank beat Alfred? What was the difference between Alfred's jump and John's?* Then near the end of the fifth grade the students are ready for three-step problems such as, *The Southern Clipper flew from Miami to Nassau in 2 hours at an average speed of 94 miles an hour. Then it flew from Nassau to Havana in 3 hours at an average speed of 83 miles an hour. How far did it fly from Miami to Havana by way of Nassau?* This logical approach to problem solving should be followed in the sixth grade by using approximate numbers, comparing numbers, using graphs and tables, and checking answers for reasonableness. In the seventh and eighth grades this logical development should be maintained.

The fifth point in my philosophy is the importance and necessity of simple vocabulary in arithmetic, especially in word problems. To this end I am advancing the thesis that the vocabulary of word problems in arithmetic should be kept one grade level below the arithmetical level in order to make it possible for the student to devote all of his arithmetic time to arithmetic and not be required to share some of it with the recognition and meaning of words in the word problems. To illustrate the waste in problem solving due to difficult vocabulary it will suffice to cite Chase's study with fourth and fifth grade students. Her test exercise included 47 words frequently found in fourth- and fifth-grade arithmetics, some of which told what a man's work is, other were about money, others about land, and still others were the names of objects to put things in. The students were asked to identify these words in terms of these four categories. From 4 to 100 per cent of the fourth-grade students and from 4 to 88 per cent of the fifth-grade students failed to iden-

tify these words correctly. The median percentage for the fourth grade was 43, and the median for the fifth grade was 20.[3]

The sixth and last point of my thesis, and the one closest and dearest to me, is the socialization and configuration of word-problem material. Many elementary school teachers have said to me, "My students are not interested in arithmetic. My largest percentage of failures and low grades are in arithmetic. If I could only find some way to stimulate interest in my arithmetic class, my percentage of low marks in arithmetic would be no greater than that in the other subjects, and my teaching tasks would be so much easier." The development of interest and motivation in arithmetic is the answer. But how? While I am not claiming to have all the answers, I am convinced that I have one of them. The best place in which to motivate these problems is to make them as nearly as possible like real life situations. The socialization and configuration of these word problems will go a long way toward making these problems live and take on real life for the students.

An examination of a random sampling of seven fourth- and fifth-grade arithmetics of relatively recent publication for the socialization and configuration of word-problem material in the treatment of "average" shows that only two of the seven texts made any attempts to present the material in configuration form. In Text A there is a page of problems captioned "Finding Averages." Each of the five problems deals with finding the average number of arithmetic examples done correctly by several students and under varying conditions. The entire page of material is centered around one central theme. The same situation holds in text B except to a greater degree. In this text there are almost two full pages of problem material, with a cut of Easter rabbits and Easter eggs, consisting of nine problems and captioned "Finding the Average." All the problems are developed around the theme of hunting Easter eggs on the farm. In this text there are also two full pages of problems, with a four-color illustration, captioned "The Gold-Fish Farm." All of the fifteen problems are centered around Father, Mother, Jane, and Jimmie buying fish at the gold-fish farm. The concept and process of finding the average were developed throughout this set of problems.

Let us contrast these two arithmetic texts with the other five in which there is no configuration of word-problem material dealing with "averages." Text C has a full page of seven problems. Problems 1 and 2 deal with walking to school, problem 3 deals with average weight, problem 4 with height, problem 5 with walking to a lake, problem 6 with homework, and problem 7 with practicing on the piano. Text D has a full page of six problems in which the average is developed. Problems 1 and 2 deal with the average scores in a game, problem 3 with a spelling contest, problems 4, 5, and 6 with an arithmetic match. Text E has two full pages on which there are eleven word problems. Two of the problems deal with fishing, two with earning money during a vacation, two with spending money, three with marks in spelling, and one with basketball scores. Text F has a full page of ten problems. One problem deals with gathering eggs, one with a basketball season, one with building a mountain highway, one with a trip on a freight boat, two with average population of towns, one with airplane speed, one with monthly earnings, one with speed of reading, and one with scores on games of ringtoss.

Obviously a set of word problems with one central theme, a unity of purpose, and a configuration of form is much more interesting and true to real life than a set of such problems that shift the student's attention and interest over eight or ten areas of living. Living is an entity, a whole, a Gestalt. Students' arithmetic experiences are more interesting and motivating when there is a unity and a configuration of them.

By way of summary, let me list my six

[3] Sarah E. Chase, "Waste in Arithmetic," *Teachers College Record*, Vol. 18, pp. 360–370.

major points, namely, (1) the introduction of arithmetic concepts and topics only when the child is psychologically prepared and ready for them; (2) practice and drill on those topics that need it and not waste time on those that have been mastered except for maintenance purposes; (3) the recognition of and provision for individual differences among pupils through diagnostic and remedial procedures; (4) a logical approach to problem solving; (5) that the vocabulary of word-problems be a grade below the arithemetical level; and (6) the socialization and configuration of word-problem material. In conclusion, let us teach that which is useful in arithmetic and teach it in a way that it can be applied and used.

Annual Meeting of the Nebraska Section of the National Council of Teachers of Mathematics, Hotel Lincoln, 147 No. 9th St., Lincoln, Neb., Saturday, May 1, 1948

PROGRAM

MORNING SESSION—10:00 A.M.

President: MISS GRACE MCMAHON
Program Chairman: MISS INEZ COOK

Preparation for Mathematics in High School

Mr. Harold C. Mardis, Principal of Lincoln High School

Postwar Trends in Mathematics

Dr. Miguel A. Basoco, Chairman, Dept. of Math., University of Nebraska

From Workshop to Classroom

Miss Maude Holden, Ord High School

LUNCHEON IN GARDEN ROOM—12:00 NOON

Guidance in Mathematics

Mr. Andrew Nelsen, Omaha Central High School

Reports on Luncheon Group Discussions

Tour of State Capitol Building

Make luncheon reservations with state representative of the Council, Miss Edith Ellis, 1724 F, Lincoln, Nebr. before April 17, 1948.

Calling All Duke Mathematics Institute "Alumni"

There will be a Duke Mathematics Institute Reunion Breakfast during the National Council of Teachers of Mathematics meeting in Indianapolis. It will be held at 8 A.M. on April 3 at the Claypool Hotel. If you plan to attend, write Veryl Schult, Wilson Teachers College, Washington, D. C., for a reservation.

Points in the Report of the Institute of Mathematical Statistics Committee on the Teaching of Statistics[1]

By HAROLD HOTELLING

The University of North Carolina, Chapel Hill, N.C.

1. Some knowledge of statistical method should be a part of a general liberal education and should be acquired by all college students.

2. Additional instruction in statistics is needed by future users of statistics such as business executives and government administrators, and still more by research workers of many kinds. For example, specialists in business research, economics, population problems, sociology, personnel selection and management, public opinion polling, biology, agriculture, metallurgy, physics and psychology need an extensive knowledge of the mathematical basis of statistical methods, since they must frequently use advanced techniques in the statistical portions of their own research, and even devise new mathematical statistical methods for their special purposes.

3. A still higher level of study of mathematical statistics, with the necessary supporting mathematics, is needed by the future teachers of statistics, including especially the teachers of nonmathematical statistics, and by research workers in the theory of statistics and professional statisticians of high grade. These constitute the smallest of the various groups, but in many respects the most crucial, since the statistical activities of the others depend ultimately on those of the creators and teachers of statistical methods and theory.

4. The organization of the teaching of statistics should be in either a Department of Statistics or in an Institute of Statistics which might [as at the University of North Carolina] include a service agency for practical statistics together with two departments, one giving instruction and carrying on research in the mathematical theory of statistics, and the other in techniques of applied statistics. Serious consideration should be given in each institution to the unification of the various statistical laboratories, and there should be a central statistical library containing all important contributions to the theory and methods of statistics possessed by the institution.

5. The first course in statistics taught each student should be in the Department of Statistics. There should in this department be two general beginning courses, one requiring calculus as a prerequisite and the other only high-school algebra. Efforts should be made to induce as many students as possible to learn calculus and then take the course requiring it. After the first course, there should be work in the application of statistical methods to economics, psychology, engineering and other fields; this work might be either in the respective departments concerned with the applications or in the Department of Statistics, according to the distribution of professorial talents in the particular institution; but all those teaching any of these courses should be well acquainted with the mathematical theory of statistics as well as with the field of application. Advanced courses in statistical theory, as well as the beginning ones, should be in the province of the Department of Statistics.

6. Teachers of statistical theory and method appointed in the future should be scholarly specialists in statistical theory

[1] Presented to the Board of Directors of the Institute at New Haven, September 4, 1947. To be published in the *Annals of Mathematical Statistics*. A further discussion of the subject with an amplification of some of the points below by Harold Hotelling is being published by the University of California Press in the volume *Symposium on Probability and Statistics.*

and method, not merely workers in one or another of the many fields in which statistics can be used.

7. Teachers of statistical theory and method should currently engage in research in these subjects, since otherwise they are in danger of soon being left behind in the rapid progress of the subject and thus getting into the position of teaching the wrong things.

8. The training of teachers of statistics should include considerable higher mathematics, with matrices and theory of functions as the extreme minimum requirement, and much more mathematics is highly desirable. Such training should also involve thorough study of the mathematical theory of statistics, together with practice in applied statistics. The time required for all this exceeds that ordinarily allotted for graduate work leading to the Ph.D. degree. A solution of the time problem may be sought in three directions:

(a) By increasing the number of years required for the doctorate. This is undesirable and practically impossible.

(b) By means of postdoctoral fellowships, or by internships in government or industry, whereby the young doctor will work under the direction of competent statisticians in applying the mathematical theory to specific practical problems. A few such internships are now available through the Graduate School of the United States Department of Agriculture. Expansion of this and similar programs is much to be desired.

(c) By getting the preliminary mathematics taught at earlier ages. The committee[2] suggests that elementary calculus be taught in high schools to students aged 17, as is done in Europe, and also that certain parts of mathematics ordinarily reserved for graduate students be taught in kindergarten and grammar school. Implementation of these suggestions will require school teachers with a knowledge of higher mathematics comparable with that of European teachers, rather than the lower levels of mathematical training that have been customary for school teachers in this country.

[2] *Members of the Committee:* Harold Hotelling (Chairman), Professor of Mathematical Statistics and Associate Director, Institute of Statistics, University of North Carolina. Dr. W. Edwards Deming, Division of Statistical Standards, Bureau of the Budget, Washington, D. C. Dr. Walter Bartky, Dean of Arts and Sciences, University of Chicago. Dr. Milton Friedman, Associate Professor of Statistics, School of Business, University of Chicago. Dr. Paul G. Hoel, Associate Professor of Mathematics, University of California, Los Angeles.

To a Pupil

By JAMES B. SPRAGUE
Bernards High School
Bernardsville, N. J.

I did not guess when first we met
That fate had been unkind,
That our association would
Produce this state of mind.
I've pled, cajoled, used cunning wiles,
I've even tried dramatics,
I wish I knew how in the world
To teach you mathematics.

Why a Label?

By Wm. S. Tobey
Junior and Senior High Schools, Long Branch, N. J.

Since only a small minority of our secondary pupils elect the so-called college-preparatory mathematics, and an even smaller minority can profit from it, why must we label all other offerings in mathematics? We offer mathematics of a general consumer type to all pupils of the ninth, tenth, eleventh, and twelfth years who wish to elect it, and for as many years as they choose to pursue it, limited only by graduation from senior high school. For those who must meet certain requirements we offer special subjects such as algebra, plane and solid geometry, and trigonometry.

Pupils electing these special subjects must meet our standards or fail. To those who elect the non-specialized course we guarantee that:

1. They will devote their time and energy to solving problems of everyday living.
2. The problems will be within their range of ability and interest.
3. They will be placed with pupils of approximately their ability in mathematics.
4. With their cooperation they will make progress. Only through manifest unwillingness to do his part can a pupil fail.

We believe it to be our task to build upon the foundation with which our pupils have been endowed, not to pass judgment on the product of creation.

Our plan is simple. Near the end of each school year standardized tests in arithmetic are administered to all pupils who elect the general type of mathematics, those who elect a special type but who, in our opinion, will find it advisable to change later, and those whose elections are such that their advisers believe they can be made to see the advisability of electing mathematics. Pupils in the ninth grade are placed on three levels on the basis of test scores. In the senior high school all pupils tested regardless of grade level are placed on seven levels on the basis of test scores. Frequently a single group may contain pupils from all three levels or years of the senior high school. Each year the pupils are re-grouped and each year the content is adjusted to the abilities of the several groups.

The plan is not a sequence in the generally accepted sense; it is, rather, a spiral with the same or similar problems appearing again and again from circuit to circuit as the pupils progress upward. On each successive attack the pupils delve more deeply into the problem; additional problems are added as the interests and abilities of the groups warrant. There are not a multitude of problems of everyday living, but a relatively limited number, and only by meeting these a number of times and from many different angles can a pupil build the proficiency necessary to insure that his mathematics will function in later life.

The extent of the spiral is determined by the abilities of the weakest and strongest pupils. The number of levels is determined, to a large degree, by the number of pupils electing the course. In case the spread of ability divided by the number of levels is too great for efficient instruction, the lower half of the group determines the basic content offered, with more difficult supplementary material provided to challenge the stronger half. Pupil interest in these groups is not less than that in college-preparatory classes, nor is the teaching more difficult, providing the teacher possesses the proper attitude.

Our course of study is as simple as our plant. The report, "Essential Mathematics for Minimum Army Need" in the October, 1943 issue of The Mathematics Teacher

serves as our guide in the selection of content, in determining the emphasis to be placed on the various items, in the method of presentation and as a criterion in the choice of texts. There are now available a number of texts that can be used almost completely and with little change in topical sequence. A larger number of texts contain valuable supplementary material.

Each year the members of the department break down the total enrollment into the several levels, choose the text best adapted to each group, determine and locate the needed supplementary material, and pool their experience and improvements in methods of teaching. This procedure insures a flexible and highly adaptive course of study.

Math Professor*

By BEATRICE SPEIGHT, *Student, University of Alabama*

I wonder if his inmost thoughts
Are organized by sets of rules,
So that he calculates his oughts
And oughtn'nts: makes his thoughts his tools?

Does he work out percentage on
The chances that B will pay,
To find himself bewildered when
His X comes out a different way?

Can he control life's variables,
Figure all the different angles,
Make events abide by tables,
Never have them end in tangles?

If he can, I'd like to know
The definitions, cubes, and squares.
There might be better ways to go
At solving my complex affairs.

Or does he find that in his actions
There's no use for rules in sets?
End with all the useless fractions
That the obtuse pupil gets?

* Contributed by Professor F. A. Lewis—Editor.

◆ THE ART OF TEACHING ◆

Approximate Square Roots

By R. C. JURGENSEN
Culver Military Academy, Culver, Ind.

THE FINAL step in the computing of a square root correct to a certain number of figures can be done by a method which is simpler than that usually presented in elementary text books. According to the usual method it is necessary, in obtaining n figure accuracy, to compute to $n+1$ figures and then round off. To find the square root of .1224 correct to tenths one writes:

$$\begin{array}{r|r} & .3\ 4 \\ \hline & .1224 \\ & 9 \\ \hline 64 & 324 \\ & 256 \\ \hline & 68 \end{array}$$

Answer .3

The question, "Isn't there a shorter way—as in long division?" is often asked. A student may be pleased to learn that there is a shorter way, one even simpler than that used in long division. By this method the answer to the above example can be given when the stage

$$\begin{array}{r} .3 \\ \hline .1224 \\ 9 \\ \hline 3 \end{array}$$ is reached.

Consider numbers lying between a^2 and $(a+1)^2=a^2+2a+1$, where a is an integer.
$(a^2+a+.25)$ has the square root $(a+.5)$
Then a number $<(a^2+a+.25)$ must have a square root $<(a+.5)$ and a number $>(a^2+a+.25)$ must have a square root $>(a+.5)$.
When the square root of $(a^2+a+.25)$ is calculated in detail the stage

$$\begin{array}{r} a \\ \hline a^2+a+.25 \\ a^2 \\ \hline a+.25 \end{array}$$ is reached.

Note the remainder $(a+.25)$

A remainder $<(a+.25)$ would imply a number $<(a^2+a+.25)$ and a square root $<(a+.5)$
A remainder $>(a+.25)$ would imply a number $>(a^2+a+.25)$ and a square root $>(a+.5)$

Then, in calculating square root when a is the integral part of the root.
a is the better approximation if the remainder $<(a+.25)$, and
$(a+1)$ is the better approximation if the remainder $>(a+.25)$.
The method can be extended to include all numbers and decimal point positions.
Several illustrative examples will be given.

Ex. 1. Find the square root of 29.16 correct to the nearest integer.
Solution:

$$\begin{array}{r} 5. \\ \hline 29.16 \\ 25 \\ \hline 416 \end{array}$$

Answer 5 (since the remainder 416 is less than 525)

Ex. 2. Find the square root of .00086 correct to hundredths.
Solution:

$$\begin{array}{r} .0\ 2 \\ \hline .000860 \\ 4 \\ \hline 460 \end{array}$$

Answer .03 (since the remainder 460 is greater than 225)

Ex. 3. Find the square root of .0624 correct to tenths.
Solution:

$$\begin{array}{r} .2 \\ \hline .0624 \\ 4 \\ \hline 224 \end{array}$$

Answer .2 (since the remainder 224 is less than 225)

Ex. 4. Find the square root of .0626 correct to tenths.

Solution:

$$\begin{array}{r|l} & .2 \\ \hline & .0626 \\ & 4 \\ \hline & 226 \end{array}$$

Answer .3 (since the remainder 226 is greater than 225)

Ex. 5. Find the square root of 5256.2501 correct to the nearest integer.

Solution:

$$\begin{array}{r|l} & 7\ 2 \\ \hline & 5256.2501 \\ & 49 \\ \hline 142 & 356 \\ & 284 \\ \hline & 722501 \end{array}$$

Answer 73 (since the remainder 722501 is greater than 722500)

Of course the last "bringing down" need not be done mechanically, and 73 is observed to be the desired number when the stage

$$\begin{array}{r|l} & 7\ 2 \\ \hline & 5256.2501 \\ & 49 \\ \hline 142 & 356 \\ & 284 \\ \hline & 72 \end{array}$$

is reached.

Aids to Teaching

By Henry W. Syer
School of Education, Boston University, Boston, Massachusetts
and
Donovan A. Johnson
College of Education, University of Minnesota, Minneapolis, Minnesota

Introduction

The following article is the first of a series which will go on as long as material and interest continue. Although the two writers listed above try to keep in touch with the latest aids to the teaching of mathematics, this department will prosper and prove its usefulness only if the teachers, administrators, librarians, commercial concerns and others who read it will keep us informed of the new developments which should be reviewed. The following items are being considered, but others should be suggested if they ought to be included: motion picture films, filmstrips, slides (2″×2″ or $3\frac{1}{4}$″×4″), instruments, models, classroom equipment, pictures (including three-dimensional), charts, free-booklets and other material, plans for construction of teaching aids, sources of materials for model construction, suggested bulletin board displays, and standardized tests.* Send us information, samples, ideas and suggestions for what you want included in this department and let us see if it can serve as a real clearing house through which mathematics teachers can exchange information about teaching materials.

Booklets

B. 1—*Fascinating Figure Puzzles*

Burroughs Adding Machine Company, Detroit, Mich.
Free booklet of puzzles, $4\frac{1}{4}$″×6″, 28 pages.

Description:

Contains forty puzzles, all old and familiar favorites, but still fascinating for those who have never seen them. Although the selection is not new, the presentation is neat and interesting. There are six line drawings to illustrate some of the puzzles. Ten pages of puzzles and eight of answers are followed by five of advertising. The name "Burroughs" is not mentioned at all on the outside of the booklet and only three times at the beginning: twice on the title page and once on the reverse of the title page.

Appraisal:

Out of the forty problems exactly half, or twenty, have some real mathematical principle behind the solution; the rest depend upon tricks, puns, plays on words or pure ingenuity to find the solution. It is well worth the time to have pupils discover which puzzles in the booklet depend upon mathematics and then display the mathematical explanation of the solution.

B. 2—*From Og to Googol*

Marchant Calculating Machine Company; Oakland 8, California, U.S.A.
Sixteen-page booklet on history of mechanical calculation. Free.

Description:

Many historical devices used to shorten or speed up arithmetic calculation are described, including the following: finger calculating, use of stones and strings and beads, the abacus, Napier's bones, Pascal's calculator, Leibnitz' reckoning machine, Babbage's folly, the Odhner machine and more modern developments. The description is written in the form of a running

* The following abbreviations will be used in titles, in order to make easier reference to objects mentioned in previous issues of this department: B—booklet, C—Chart, D—bulletin board display, E—equipment, F—film, FS—film-strip, I—instrument, M—model, P—picture, PC—plans for construction, S—glass slide, SL—sources of material for laboratory work, T—standardized tests.

story and there are thirteen line drawings to illustrate it.

Appraisal:

The material is compact, accurate and interesting. Two pages are devoted to covers, one to a preface, ten to historical material and three to historical material with an advertising slant. The book is out of proportion in the direction of modern, mechanical machines and slights, consequently, the older, more primitive methods. However the illustrations are excellent and the text should prove the basis for many essays, projects and replicas of older, computing devices.

Charts

C. 1—*Navigation Chart*

Air-Age Education Research, 80 East 42nd Street, New York, New York
Colored poster-chart; 35″×47″; $1.00

Description:

This large attractive chart has a picture in the center showing how cross-country trips are plotted on charts. Surrounding it are illustrations showing various types of navigation: contact flight, dead reckoning, radio navigation and celestial navigation; also illustrations explaining features of maps and pictures of navigational instruments: aviation clock, altimeter, magnetic compass and air speed indicator.

Appraisal:

The pictures are authentic, sufficiently large and quite attractive. There is no doubt that interest in aviation and its uses of mathematics is very great; moreover, some who take mathematics will find their chief use of it in aviation. The cost is not prohibitive for the material. Such a chart could be made part of a larger display of applications of mathematics. It could be supplemented with pupil-made charts using the principles of navigation discussed. Actually the usefulness of the chart is greatly increased by the booklet on *Navigation* also furnished by Air-Age Research.

Equipment

E. 1—*Matho-O-Felt* (Plane Geometry Device)

Mr. P. E. Huffman, Hutsonville, Illinois
19 felt figures, 1 felt base, 1 set of 10 felt letters, manual. Price $20.00 plus transportation.

Description:

The felt background may be attached to a vertical bulletin board and the other pieces, in the shapes of geometric figures, placed upon it. These stick to the background by pressure alone; no tacks or adhesives are used. Felt letters can be used to identify parts of the figures. Figures may be placed on top of each other, or overlapping to any desired degree, and they will still adhere. The figures are supplied in several colors for contrast, attractiveness and ease in identification.

Appraisal:

The idea is certainly clever and workable. It does have its limitations: areas and whole or parts of figures are easily identified, lines and angles in figures are not so easily designated. The speed with which figures can be built up and manipulated with obvious maintenance of size and shape is helpful, but the limitations to certain, common diagrams is harmful. The cost seems large, both for the amount of material supplied and for the amount of use it would receive. Why not use the same idea and have the pupils cut diagrams out of felt or even flannel and make their own devices?

The greatest use will be in discussions of plane geometry theorems and originals concerning superposition, composition and decomposition of areas, and equal but noncongruent areas.

Films

F. 1—*Story of Money*

International Film, Inc., 84 East Randolph St., Chicago 1, Ill.
2 reels, 16mm. sound, Black and white

Content:

This film tells the history of money and our banking processes. Beginning with primitive man, it shows how barter created a need for objects of value to assist in bargaining. After using ornaments and precious metals, man formed the metals into coins, but due to lack of uniformity the value of the coins depended upon their weight. Since there were few sources of investment, goldsmiths became increasingly numerous. To expedite the maintenance of accounts, receipts were issued. These receipts were used to settle debts and soon changed into our modern system of checks. This created a need for clearing houses and government control of banking and minting of coins. The film concludes by emphasizing that wealth depends upon resources not upon the coins in circulation, that money is merely a convenient means of exchange, and that checks have considerable effect on production and business efficiency.

Appraisal:

This British film is a professional production of an important and complex subject. It gives a quick summary of the basis for our banking system. Its discussion of the value of money is appropriate. However, this is a story that needs more complete coverage. It should be supplemented by additional films on the entire processes of banks and the role of government regulation of banks. It is an informational film that is well worth showing in a unit on money or banking.

Technical Qualities:
Photography: Very good
Sound: Excellent
Commentary: Appropriate and clear
Content: Excellent
Level: Junior and senior high school

F. 2—*Property Taxation*
Encyclopaedia Britannica Films
20 North Wacker Drive, Chicago, Ill.
Educational Advisor: H. F. Alderfer
1 reel, 16 mm. Black and white, sound

Content:

This film shows for what taxes are used and how the tax rates are determined. It illustrates the community services that are paid for by taxes; namely, police and fire protection, schools, and playgrounds. It discusses the issuing and selling of bonds to pay for current expenses as well as permanent improvements. Problems are worked to show the annual and semi-annual interest payments on these bonds and to show how much tax money must be set aside for bond retirement and interest payments.

In developing the basis for tax rates, the film discusses the kinds of property which are not taxable; namely, schools, churches, and government property. It shows city, county, and school board officials setting the tax rate by comparing the budget of expenditures with the total assessed valuation. Problems are worked to show how the different methods of expressing the tax rate can be used to determine the tax to be paid on a home.

Appraisal:

This film will be welcomed by teachers of seventh and eighth grade arithmetic who wish to emphasize the social phase of arithmetic instruction. Although problems are illustrated and solved in this film, they could be explained equally as well by the teacher. However, the emphasis on the public services afforded by taxes is well done by the pictures of these community activities. Relating the problems to the life situation and showing the common technique used by authorities should make the instruction more functional. Of course an excursion through the community to see these same activities would be more realistic, in which case this film could be used as a preparation for such an excursion.

Technical qualities:

Photography: Very good
Sound: Very good
Commentary: Appropriate but not unusually interesting.

F. 3—*Rectilinear Coordinates*
Knowledge Builders
RKO Building, Radio City, New York, N. Y.
Educational Advisor: John H. Lewis
Drawings by McCrory Studio
1 reel, 16 mm. sound Black and white, 1942

Content:

This film shows the use of coordinates to locate points in space. It opens with a historical note on the invention of coordinates by DesCartes. Beginning with a point as zero-dimensional and a line as one-dimensional, the film illustrates how one coordinate will locate a point on a line, two coordinates will locate a point on a plane, and three coordinates will locate a point in space. The relationship between these coordinate systems is shown by a moving point generating a line, a moving line generating a plane, and a moving plane giving three-dimensional space. The change in value of coordinates is shown very clearly as the location of a point changes. In this manner it is possible to demonstrate the different ways in which coordinates change when action is parallel to a line, parallel to a plane, motion in a plane, or in any direction. Scaled axes are used to show how the coordinates are determined in each system. In the three-dimensional system the film shows distances of points from the three planes determined by the three axes.

Appraisal:

Although many of the things done by this picture could be done by blackboard demonstration, its illustration of the three-dimensional coordinate system and of the change in coordinates of a point as the point moves should contribute much to the student's understanding of cartesian coordinates. Due to the fact that the beginning of the film presents elementary facts and the end more advanced concepts, it is difficult to indicate a level for which the film is entirely appropriate. The commentary is clear but the introduction of many new terms, such as *octant*, needs to be planned for prior to showing the film. The changes of coordinates are sometimes very rapid and the negative signs are not very distinct.

Technical Qualities:

Photography: Good drawings but the axes are very wide lines
Sound: Very good
Content: Covers too great a range of material

Film-Strips

FS. 1—*Timekeepers Through the Ages*
Visual Sciences
Suffern, N. Y.
Film-strip, 41 frames, 1939

Content:

This film-strip shows the development of devices for telling time from primitive man to the modern electric clock. It shows such timekeeping devices as the shadow pole, chinese water clock, candle, knotted rope, oil lamp, sun dial, hour glass, pendulum clocks, spring watches, metronome, cuckoo clocks and electric clocks. It shows the various time belts and world time determined by astronomical clocks. It concludes by discussing the uses of clocks to regulate traffic, signals, beacons, lights, and household appliances.

Appraisal:

The content of this film-strip is well chosen and very appropriate for mathematics classes. It is unfortunate that the drawings are like those in ancient textbooks and, thus, lose some interest.

Technical rating:

Photography: Fair
Content: Very good
Level: Junior high school mathematics

Instruments

I. 1—Inexpensive Slide Rules
Lawrence Engineering Service, 50 Smith Street, Peru, Indiana
Slide rules ranging in price from \$.50 to

$1.00 for ordinary calculations and also for specialized calculating jobs.

Description:

Ordinary Rule: Contains A, B, CI, C, D and K scales. Will do multiplication, division, squares, cubes, square and cube roots. 10″ long. $.50.

Multiphase: Same scales on face as ordinary rule with CI calibrations in red. Back of slide contains S, L and T scales. Extends use of rule to trigonometric functions. Comes in flexible case. $1.00.

Instruction booklet: 32 pages. Satisfactory but frail booklet. Will not stand up under hard usage. $.10.

Specialized slide rules at $.50. Lumber calculator, copper wire selector, photoengraver's proportion calculator, printer's proportion rule, music transposer, and codemaker set.

Specialized rules at $1.00. Cutting speed calculator, copyfitter, pricing and inventory rule, and photography flashrules.

Appraisal:

The ordinary and multiphase slide rules meet the need of classes which are learning slide-rule operation. They provide the extremely necessary supplements to the large demonstration slide-rule usually used. The prices are low enough so that a rule can be given to each pair of students for class-room exercises. The instruction booklet is well written, but not substantial enough to use as a textbook. It would soon be worn out. Mimeographed exercises together with class explanation would be more useful in classroom teaching. The various types of specialized slide rules would be useful only as examples of how the slide rule principle can be applied to many, varied tasks of computation which business and industry require. A few selected from those available could be owned by the mathematics department, or on display in the mathematics laboratory or museum.

The workmanship on these rules is much better than the price would lead you to expect. In some cases the wood of which they are made has been known to shrink or warp and so cause the slide to bind or the scales to become distorted. This tendency is not general enough to prevent them from being very useful. The calibrations are clear and accurate, and the general construction is sturdy and yet light in weight.

Pictures

P. 1—*Visual Mathematics* (Algebra Pictures)

Miss Ida D. Fogelson, 5520 South Shore Drive, Chicago, Illinois

Four pictures, $8\frac{1}{2}'' \times 11''$, in color to illustrate axioms of algebra. $.50.

Description:

These four pictures illustrate the four fundamental axioms of algebra. For example, "If equals are added to equals the results are equal" is illustrated by two boys (same size) having a tug of war and being helped by two dogs (also the same size). Subtraction is shown by girls hanging from a pivoted see-saw; multiplication by two sets of nine cats each; and division by a woman breaking each of two candy sticks in two to give to two little girls.

Appraisal:

The pictures are very attractively printed in bright and clear colors. The type of illustration used is direct and at the level of the pupil-audience to which it is directed. All but multiplication are well chosen examples which clarify the principle at a glance.

The greatest use of these pictures is to show teachers and pupils alike what simple, useful illustrations can be designed to make mathematical principles concrete. They should serve as an inspiration to cause classes to find and create other, similar, more personal examples; for it is in the creation of such pictures, not in their use, that the greatest value lies. They should certainly be seen by each teacher and each class at least once. Why not make them a

part of the permanent mathematics teaching equipment of each building?

Plans For Construction

PC. 1—*Oriental Abacus*
Industrial Arts Cooperative Service, 519 West 121st Street, New York, New York
Booklet of directions; mimeographed; 6 pages; $.25.

Description:

A short description and drawing of an abacus is followed by directions for counting, adding and subtracting. Finally there is a set of directions in twelve simple steps for constructing an abacus; this is accompanied by an exploded drawing showing the method.

Appraisal:

There is nothing in this booklet that cannot be obtained in more detail from other sources, but it is very convenient to have it in one place in a fairly simple form. For anyone with a little mechanical skill the directions for putting together the pieces of wood and beads to make the model are unnecessary, but most people prefer to have definite steps laid out for them.

EDITORIALS

Guidance Pamphlet in Mathematics

THE FOLLOWING letter from Professor Esther Lloyd-Jones of Teachers College, Columbia University, concerning the Guidance Pamphlet in Mathematics will be of interest to our readers:

> I am fascinated with the Guidance Report of the Commission on Post-War Plans. The material contained will certainly be of the most practical value to counselors. The dramatic way in which the information is presented makes it one of the most readable reports of this sort I have seen.
>
> The very best feature of this article is that mathematics teachers themselves will read the journal in which the report appears and certainly should be able to give their students much more practical help and motivation as a result.
>
> Congratulations to you on having had the opportunity to publish it.

The New Jersey Association of Mathematics Teachers has ordered 1,000 copies. the New York City Chairman of Mathematics 500 copies, the Mathematical Association of America 500 copies, and so on for several large groups of mathematics teachers. How many copies will your group need? We now have orders for over 10,000 copies; so we have reprinted 25,000 copies which are now ready at 25¢ each postpaid, or if you order 10 or more 10¢ each postpaid. Send your orders in before the supply is exhausted.—W.D.R.

Satisfied Customers

WE ALL know that many pupils in the secondary school not only do not obtain a proper understanding of the mathematics which is offered them, but fail the subject at the end of a particular course. This is bad not only for the pupil but also for the subject because many such pupils, and even some of those who receive passing marks, develop a life-long hatred of the subject, particularly in algebra. Moreover, the pupils who fail spend too much time in failing and this only makes the situation worse. Since the high school population has increased from approximately 500,000 in 1900 to 7,000,000 in 1948 it is clear that the great range in individual differences in native ability, experiences, and interests must be taken into consideration in any curriculum revision program. In mathematics we must begin at once to place more emphasis upon the need for a two or perhaps even a three track system in mathematics to meet the varying needs of high school pupils. In addition the content material for all pupils should be reconsidered and improvements made where failure to do so will bring mathematics into further disfavor.—W.D.R.

Emma Viola Hesse

MISS EMMA VIOLA HESSE, a member of the Board of Directors of the National Council of Teachers of Mathematics, passed away on December 31, 1947. She left a mother, Stella Hesse, and five sisters.

Miss Hesse was born in Boulder Creek California, 61 years ago. She graduated from the University of California in 1910 and, after a year of graduate study, she spent the next 14 years teaching in the Petaluma High School. She then spent a year of further graduate study at Teachers College, Columbia University, where she received the master's degree. Then followed 20 years of teaching in Oakland.

Miss Hesse served the cause of youth and mathematics long and well. She was a superb teacher and was devoted to her pupils and her work. Moreover, she had a wonderful personality—and was loved by all who knew her.—W.D.R.

IN OTHER PERIODICALS

By Nathan Lazar

Midwood High School, Brooklyn, New York

The American Mathematical Monthly

August–September 1947, Vol. 54, No. 7
1. Mac, Mark, "Random Walk and the Theory of Brownian Motion," pp. 369–391.
2. Pinney, Edmund, "Vibration Modes of Tapped Beams," pp. 391–394.
3. Thébault, Victor, "Tetrahedrons Having a Common Face," pp. 395–398.
4. "Instruction and Research in Applied Mathematics," pp. 398–399.
5. Mackey, G. W., "The William Lowell Putnam Mathematical Competition," pp. 400–403.
6. Mathematical Notes, pp. 404–406.
   a. Chittenden, E. W., "On the Number of Paths in a Finite Partially Ordered Set."
   b. Kasner, Edward, and DeCicco, John, "Note on Conjugate Harmonic Functions."
7. Classroom Notes, pp. 407–411.
   a. Oakley, C. O., "End-point Maxima and Minima."
8. Letters to the Editor, pp. 409–411.
   a. Miley, M. F., "The Trial Integral Method."
   b. Boyer, C. B., "The Equation of an Ellipse."
   c. Jones, P. S., "The Remainder Theorem."
   d. Johnson, R. A., "Linear Differential Equations."
9. Elementary Problems and Solutions, pp. 412–417.
10. Advanced Problems and Solutions, pp. 418–423.
11. Recent Publications, pp. 423–427.
12. Clubs and Allied Activities, pp. 427–429.
13. News and Notes, pp. 430–435.
14. Official Reports and Communications of *The Mathematical Association of America.*

October 1947, Vol. 54, No. 8
1. A Distinguished Contributor to the *Monthly*, pp. 443–446.
   a. Bryne, Col. W. E., "Victor Thébault—The Man."
   b. Starke, E. P., "Thébault—The Number Theorist."
   c. Court, A. N., "Thébault—The Geometer."
2. Thébault, Victor, "Concerning the Euler Line of a Triangle," pp. 447–453.
3. Brown, O. E., and Nassau, J. J., "A Navigation Computer," pp. 453–458.
4. Phipps, C. G., "The Jeep Problem: A More General Solution," pp. 458–462.
5. Mathematical Notes, pp. 462–464.
   a. Hamming, R. W., "Subseries of a Monotone Divergent Series."
   b. Ankenny, N. C., "One More Proof of the Fundamental Theorem of Algebra."
6. Classroom Notes, pp. 465–470.
   a. Lubin, C. I., "Differentiation of the Trigonometric Function."
   b. Hummel, P. M., and Seebeck, Jr., C. L., "A Treatment of Bonds between Interest Dates."
7. Elementary Problems and Solutions, pp. 471–478.
8. Advanced Problems and Solutions, pp. 479–491.
9. Recent Publications, pp. 491–496.
10. Clubs and Allied Activities, pp. 497–500.
11. News and Notices, pp. 501–509.
12. Official Reports and Communications of the *Mathematical Association of America*, pp. 509–514.

Bulletin of the Kansas Association of Teachers of Mathematics

April 1947, Vol. 21, No. 4
1. Klooster, H. J., "Veterans in Mathematics Classes at the University of Colorado," pp. 51–53.
2. Kneale, Sam, "On Non-Euclidean Planes," pp. 53–55.
3. "Material for Introducing Hyperbolic Functions," p. 55.
4. Sanger, R. G., "The History of Geometry—I," pp. 56–58.
5. Fawcett, Harold, "Unifying Concepts in Mathematics," pp. 59–62.

October 1947, Vol. 22, No. 2
1. Price, G. Bailey, "Careers in Mathematics," pp. 3–10.
2. "Mathematics Requirements for Entrance to Engineering Schools at the University of Kansas and at Kansas State College."
3. Miss Holroyd Retired from Teaching at Kansas State College.
4. Betz, William, "Functional Competence in Mathematics."
5. Gillam, B. E., "Mathematics for Today."

The Mathematical Gazette

July 1947, Vol. 31, No. 295.
1. "*The Ground of Artes*," p. 129.
2. Piggott, H. E., and Steiner, A., "Isogonal Conjugates," pp. 130–144.
3. Brown, B. M., "Solution of Differential Equations by Operational Methods," pp. 145–153.
4. Hargest, T. J., "Optics and Electron Optics," pp. 154–163.
1. "Mathematical Notes," pp. 164–173.
   1966. Haskell, H. N., "On the Solution of a Triangle Given Two Sides and the Included Angle.
   1967. G. L. G., "Note on Continuity."
   1968. R. L. G., "A Note on Identities."
   1969. Lockwood, E. H., "Volume to a Given Depth of a Cylindrical Tank with Spherical Caps as Ends."
   1970. Phillips, E. G., "On the Positive and Negative Sides of a Line."
   1971. E. H. N., "The Brocard Angle."

1972. Picken, D. K., "On Notes 1727 and 1842, (with 605): Extension of Simson's Line."
1973. Mason, J. I., "On Euclid VI. 3."
1974. Parameswaran, S., "Right Circular Cone."
1975. Vajda, S., "Shortcuts in Multiplication on a Calculating Machine."
1976. F. G. M., "Why Does a Bicycle Keep Upright?"

6. Reviews, pp. 174–192.

School Science and Mathematics
October 1947, Vol. 47, No. 7

1. Carter, Pane D., "From a Mechanistic to a Meaningful Program of Arithmetic Instruction: A Suggested Approach," pp. 604–608.
2. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 649–652.
3. Haner, Wendall W., "Teaching the Subtraction of Signed Numbers," pp. 656–658.
4. Problem Department, pp. 659–663.
5. Books and Pamphlets Received, pp. 664–667.
6. Book Reviews, pp. 668–676.

November 1947, Vol. 47, No. 8.

1. Chrisman, E. B., "Let's Examine Our Tests," pp. 684–686.
2. Carnahan, Walter H., "Geometric Solutions of Quadratic Equations," pp. 687–692.
3. DeMilt, Clara, "The Origins of Our Numerical Notation," pp. 701–708.
4. Meighan, John N., "Methods of Solving Elementary Systems of Equations in Two Unknowns," pp. 709–714.
5. Simer, Door M., "Slide Rule Instruction for Students of High School Chemistry," pp. 725–728.
6. Fox, Morley F., "A Geometrical Discussion of the Gravitational Laws of Inverse Squares," pp. 739–742.
7. Utz, W. Roy, "The Conic Sections and the Solar System," pp. 742–744.
8. Tan, St. Kaidy, "A New Method for Extraction of Algebraic Square Root," pp. 745–746.
9. Problem Department, pp. 747–751.
10. Books and Pamphlets Received, pp. 751–754.
11. Book Reviews, pp. 754–760.

The National Council of Teachers of Mathematics Twenty-Sixth Annual Meeting

Claypool Hotel, Indianapolis, Indiana, April 1, 2, and 3, 1948

PROGRAM

THURSDAY, APRIL 1, 1948

7:30 P.M. Meeting of the Board of Directors of the National Council of Teachers of Mathematics. Parlor E

FRIDAY, APRIL 2, 1948

10:00 A.M. to 12:00 Noon. Elementary School Section. Chateau Room.

Presiding: Ben A. Sueltz, State Teachers College, Cortland, New York

Future of Arithmetic in the Activity or Pupil-Center-of-Interest Type of School
Lois Knowles, University of Missouri, Columbia, Missouri

Future of Arithmetic in the School with Subject-Matter Organization
Irene Sauble, Public Schools, Detroit, Michigan

A Summary of Research and Investigations and their Implications on the Organization and learning of Arithmetic
H. Van Engen, Iowa State Teachers College, Cedar Falls, Iowa

Discussion
Olive Wear, Public Schools, Fort Wayne, Indiana

10:00 A.M. to 12:00 Noon. Senior High School Section. Assembly Hall

Presiding: H. W. Charlesworth, East High School, Denver, Colorado

Problems in Algebra Versus Problems in Life
Philip Peak, Indiana University, Bloomington, Indiana

Using Special Interests to Stimulate the Study of Mathematics
Ida Mae Heard, Southwestern Louisiana Institute, Lafayette, Louisiana

For Mathematics: A Reprieve or Pardon?
Daniel W. Snader, University of Illinois, Galesburg, Illinois

Coordinating High School and College Mathematics
Aubrey J. Kempner, University of Colorado, Boulder, Colorado

Discussion

10:00 A.M. to 12:00 Noon. Junior College Section. Tower Room

Presiding: Lee E. Boyer, State Teachers College, Millersville, Pennsylvania

Current Problems in the Teaching of the First Two Years of College Mathematics
Phillip S. Jones, University of Michigan, Ann Arbor, Michigan

The Use of Demonstration Devices in the Teaching of Freshman College Mathematics
Edwin L. Godfrey, Defiance College, Defiance, Ohio

The Unit Test in Junior College Mathematics
Charles W. Moran, Wright Junior College, Chicago, Ill.
Minimum Mathematical Essentials for College Graduates
F. W. Kokomoor, University of Florida, Gainesville, Florida
12:00 Noon. Luncheon for Delegates and State Representatives. Empire Room

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2:00 to 3:30 P.M. Junior High School Section. Tower Room
Presiding: Geraldine Kauffman, Public Schools, East Chicago, Indiana
The Philosophy of Education which Demands Better Understanding in Arithmetic
Virgil Mallory, State Teachers College, Montclair, New Jersey
An Analysis of Basic Principles in Arithmetic and General Mathematics which Are Found in all Problem Situations
Guy T. Buswell, University of Chicago, Chicago, Illinois

2:00 to 3:30 P.M. Teacher Training Section. Parlor B
Presiding: Veryl Schult, Wilson Teachers College, Washington, D. C.
Mathematics in the Report of the President's Scientific Research Board
Raleigh Schorling, University of Michigan, Ann Arbor, Michigan
Dramatic Incidents in the Learning of Plane Geometry
Frances Burns, High School, Oneida, New York
In and Around Mathematics
W. W. Rankin, Duke University, Durham, North Carolina

2:00 to 3:30 P.M. Guidance Section. Assembly Hall
Presiding: Walter H. Carnahan, D. C. Heath and Company, Boston, Massachusetts
Materials Available for the Mathematics Guidance Program
K. Eileen Beckett, Lebanon High School, Lebanon, Indiana
The Organization of a Mathematics Guidance Program
F. Lynwood Wren, George Peabody College for Teachers, Nashville, Tennessee
Discussion

4:00 to 5:30 P.M. Reception. Chateau Room. Members of the Council are guests of the Indiana Council of Teachers of Mathematics and the Indianapolis Mathematics Club

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7:00 to 8:00 P.M. Conference on Coordinating High School and College Mathematics. Parlor B

8:00 to 10:00 P.M. General Meeting. Assembly Hall
Presiding: Walter H. Carnahan, D. C. Heath and Company, Boston, Massachusetts
Address of Welcome
Myrl H. Ahrendt, President of the Indiana Council of Teachers of Mathematics, Anderson College, Anderson, Indiana
Response
Carl N. Shuster, President of the National Council of Teachers of Mathematics, State Teachers College, Trenton, New Jersey
Teaching Mathematics as a Guide to Practical Living
Will E. Edington, DePauw University, Greencastle, Indiana
Mathematics for the Other Eighty-five Per Cent
William A. Gager, University of Florida, Gainesville, Florida
A Teacher of Mathematics
Howard F. Fehr, State Teachers College, Montclair, New Jersey
High Points in the Reports of the Commission on Post-War Plans.
Speakers: William Betz, Raleigh Schorling, F. Lynwood Wren. Time, 15 minutes.

SATURDAY, APRIL 3, 1948

8:30 to 9:00 A.M. General Business Meeting of the National Council of Teachers of Mathematics. Assembly Hall

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9:00 to 9:45 A.M. Meeting of the Board of Directors of the National Council of Teachers of Mathematics. Parlor E

9:00 to 9:45 A.M. General Business Meeting of the Indiana Council of Teachers of Mathematics. Assembly Hall

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10:00 A.M. to 10:00 Noon. Junior High School Section. Florentine Room
Presiding: Philip Peak, Indiana University, Bloomington, Indiana
Specific Illustrations of the Use of Aids in Understanding Junior High School Mathematics.
The Blackboard Sketch
Mary Potter, Public Schools, Racine, Wisconsin
Difficulties in Reading
Lenore John, University of Chicago, Chicago, Illinois
Three-Dimensional Materials
Alice Rose Carr, Ball State Teachers College, Muncie, Indiana
Group Discussion
Elda L. Merton, Public Schools, Waukesha, Wisconsin
The Future of Slides and Films for Junior High School Mathematics
Henry W. Syer, Boston University, Boston, Massachusetts
School and Life Situations of the Pupil
Beatrice Barnes, Kramer Junior High School, Washington, D. C.

10:00 A.M. to 12:00 Noon. Senior High School Section. Chateau Room
Presiding: George E. Hawkins, Lyons Township High School and Junior College, La Grange, Illinois
Mathematics for All Students in High School
Gene S. McCreery, Ball State Teachers College, Muncie, Indiana
Notes from a Mathematics Classroom
Joseph A. Nyberg, Hyde Park High School, Chicago, Illinois
Functional Competence in Mathematics: Its Meaning and Attainment
William Betz, Rochester, New York
Report from Conference on Coordinating High School and College Mathematics
Discussion

10:00 A.M. to 12:00 Noon. Teacher Training Section. Parlor B
Presiding: L. Harper Whitcraft, Ball Teachers College, Muncie, Indiana
A Basic Course in Mathematics for Teachers
Walter O. Shriner, Indiana State Teachers College, Terre Haute, Indiana
The Use of Multi-Sensory Aids in the Training of Teachers
Howard F. Fehr, State Teachers College, Montclair, New Jersey
Teaching is Fun
John R. Clark, Teachers College, Columbia University, New York, New York

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10:00 A.M. to 12:00 Noon. Film Section. Assembly Hall
There will be a continuous showing of the latest slides and films of interest in the teaching of mathematics
Arranged through the courtesy of the Bureau of Audio-Visual Aids, Indiana University, Bloomington, Indiana

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12:00 Noon. Annual Discussion Luncheon Riley Room

2:00 to 3:30 P.M. Elementary School Section. Florentine Room
Presiding: Inez Morris, Indiana State Teachers College, Terre Haute, Indiana
The What and Why of Meaningful Arithmetic in the Elementary Grades
William A. Brownell, Duke University, Durham, North Carolina
Neglect of Meaning in Elementary Arithmetic Reflects on Junior High Learning
Eugene Smith, Ohio State University, Columbus, Ohio
Discussion
Joy Mahachek, State Teachers College, Indiana, Pennsylvania

2:00 to 3:30 P.M. Multi-Sensory Aids Section. Chateau Room
Presiding: Miles C. Hartley, University of Illinois, Urbana, Illinois
Supplementing the Geometry Textbook with Visual Aids
Rachel P. Keniston, Stockton High School, Stockton, California
A Study of the Relative Effectiveness of Films and Filmstrips in Teaching Geometry
Donovan A. Johnson, University of Minnesota, Minneapolis, Minnesota
A Method of Using Flexible Devices in the Teaching of Geometry
John F. Schacht, Bexley High School, Columbus, Ohio

2:00 to 3:30 P.M. Film Section. Assembly Hall

There will be a continuous showing of the latest slides and films of interest in the teaching of mathematics

Arranged through courtesy of the Bureau of Audio-Visual Aids, Indiana University, Bloomington, Indiana

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6:30 P.M. Annual Banquet. Chateau Room

Toastmaster: W. D. Reeve, Teachers College, Columbia University

Address: Harold P. Fawcett, Ohio State University, Columbus, Ohio. The Development of a Mathematics Teacher.

DISCUSSION LEADERS

Annual Discussion Luncheon

Riley Room, Claypool Hotel
April 3, 1948, 12:00 Noon

At the Discussion Luncheon there will be a leader and a specific topic for discussion at each table. Guests are seated at the tables according to the discussion in which they wish to participate. The price per plate (including gratuities) is $1.65. Reservations, with check and first, second, and third choice of discussion leader, should be sent by March 20 to Ada M. Coleman, Manual Training High School, Indianapolis 4, Indiana.

Those who have made reservations will receive their tickets at the Registration Desk on the Mezzanine Floor.

1. Providing for the Exceptional Pupils
   George E. Hawkins, Lyons Township High School and Junior College, La Grange, Illinois
2. What Pupils Should Take Eleventh and Twelfth Grade Mathematics?
   Ona Kraft, Collinwood High School Cleveland, Ohio
3. Operational Techniques and Meanings
   Harold Fawcett, Ohio State University, Columbus, Ohio
4. Mathematics in the Guidance Program of the Secondary School
   F. L. Wren, George Peabody College for Teachers, Nashville, Tennessee
5. Jokes Pertaining to Mathematics
   Ida Mae Heard, Southwestern Louisiana Institute, LaFayette, Louisiana
6. Salvaging Mathematics for the Non-Academic Student
   Mary Potter, Public Schools, Racine, Wisconsin
7. Opportunities for Training in Generalization in Elementary Algebra
   Inez Morris, Indiana State Teacher-College, Terre Haute, Indiana
8. Tricks of the Trade
   Robert Belding, Arsenal Technical Schools, Indianapolis, Indiana
9. The Required Mathematics Course for Teachers Colleges
   Lee Boyer, State Teachers College, Millersville, Pennsylvania
10. What Instructional Aids Are Being Used and Are Needed for the Teaching of Algebra?
    E. H. C. Hildebrandt, Northwestern University, Evanston, Illinois
11. Aids to Better Teaching
    H. W. Charlesworth, East High School, Denver, Colorado
12. What's the Matter with Mathematics?
    W. D. Reeve, Teachers College, Columbia University, New York, New York
13. How Can We Make Our Second Track Courses in Mathematics More Respectable?
    Veryl Schult, Wilson Teachers College, Washington, D. C.
14. My Favorite Problem and Why I Like It
    Maurice Hartung, University of Chicago, Chicago, Illinois
15. Teaching Locus in Plane Geometry
    Virgil Mallory, State Teachers College, Montclair, New Jersey
16. How Should the Guidance Pamphlet Be Used?
    Raleigh Schorling, University of Michigan, Ann Arbor, Michigan
17. College General Mathematics for Non-Mathematics Majors
    Houston T. Kurnes, University of Louisiana, Baton Rouge, Louisiana
18. Senior High School General Mathematics
    Edna M. Norskog, Alexandria, Minnesota
19. An Exchange of Experiences in the Teaching of Non-College-Preparatory Mathematics
    A. Brown Miller, Shaker Heights, Cleveland, Ohio
20. What Should Be the Basis of Mathematics Training for All Pupils?
    E. R. Breslich, University of Chicago, Chicago, Illinois

21. Some Questions Related to Trigonometry in High School
    Charles H. Butler, Western State Teachers College, Kalamazoo, Michigan
22. What Can We Do For Superior Students in Junior High School?
    Alice Rose Carr, Ball State Teachers College, Muncie, Indiana
23. What Shall Be the Nature of a Two-Track Program in Secondary Mathematics?
    William Betz, Rochester, New York

COMMITTEES

1. Program Committees

Elementary School

Inez Morris, Indiana State Teachers College, Terre Haute, Indiana, Chairman
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Lois Knowles, University of Missouri, Columbia, Missouri
Irene Sauble, Public Schools, Detroit, Michigan
H. Van Engen, Iowa State Teachers College, Cedar Falls, Iowa
Olive Wear, Public Schools, Fort Wayne, Indiana
William A. Brownell, Duke University, Durham, North Carolina
Eugene Smith, Ohio State University, Columbus, Ohio
Joy Mahachek, State Teachers College, Indiana, Pennsylvania

Junior High School

Philip Peak, Indiana University, Bloomington, Indiana, Chairman
Geraldine Kauffman, Public Schools, East Chicago, Indiana
Virgil Mallory, State Teachers College, Montclair, New Jersey
Guy T. Buswell, University of Chicago, Chicago, Illinois
Mary Potter, Public Schools, Racine, Wisconsin
Lenore John, University of Chicago, Chicago, Illinois
Alice Rose Carr, Ball State Teachers College, Muncie, Indiana
Elda L. Merton, Public Schools, Waukesha, Wisconsin
Henry W. Syer, Boston University, Boston, Massachusetts
Beatrice Barnes, Kramer Junior High School, Washington, D. C.

Senior High School

H. W. Charlesworth, East High School, Denver, Colorado, Chairman
George E. Hawkins, Lyons Township High School and Junior College, La-Grange, Illinois
J. E. Potzger, Butler University, Indianapolis, Indiana
Philip Peak, Indiana University, Bloomington, Indiana
Ida Mae Heard, Southwestern Louisiana Institute, LaFayette, Louisiana
Daniel W. Snader, University of Illinois, Galesburg, Illinois
Aubrey J. Kempner, University of Colorado, Boulder, Colorado
Gene S. McCreery, Ball State Teachers College, Muncie, Indiana
Joseph A. Nyberg, Hyde Park High School, Chicago, Illinois
William Betz, Rochester, New York

Teacher Training

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Raleigh Schorling, University of Michigan Ann Arbor, Michigan
Frances Burns, High School, Oneida, New York
W. W. Rankin, Duke University, Durham, North Carolina
Walter O. Shriner, Indiana State Teachers College, Terre Haute, Indiana
Howard F. Fehr, State Teachers College, Montclair, New Jersey
John R. Clark, Teachers College, Columbia University, New York, New York

Multi-Sensory-Aids

E. H. C. Hildebrandt, North Western University, Evanston, Illinois, Chairman
Miles C. Hartley, University of Illinois, Urbana, Illinois
Rachel P. Keniston, Stockton High School, Stockton, California
Donovan A. Johnson, University of Minnesota, Minneapolis, Minnesota
John F. Schacht, Bexley High School, Columbus, Ohio

Guidance

Walter H. Carnahan, D. C. Heath and Company, Boston, Massachusetts, Chairman
K. Eileen Beckett, Lebanon High School, Lebanon, Indiana

F. Lynwood Wren, George Peabody College for Teachers, Nashville, Tennessee

Junior College

E. H. C. Hildebrandt, Northwestern University, Evanston, Illinois, Chairman
Lee E. Boyer, State Teachers College, Millersville, Pennsylvania
Phillip S. Jones, University of Michigan, Ann Arbor, Michigan
F. W. Kokomoor, University of Florida, Gainesville, Florida

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Philip Peak, Indiana University, Bloomington, Indiana

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MISCELLANEOUS INFORMATION

National Council Headquarters . . Parlor B
Exhibits . . . . Mezzanine Floor
Registration . . . . Mezzanine Floor

Reservations for the luncheons and the Banquet should be made by March 20. The price of the luncheons is $1.65, and for the Banquet, $2.75. These prices include all gratuities. Reservations should be sent to Ada M. Coleman, Manual Training High School, Indianapolis 4, Indiana. Those who have made reservations will receive their tickets at the Registration Desk on the Mezzanine Floor.

Reservations for hotel rooms should be sent directly to the Claypool Hotel, Indianapolis, Indiana. The hotel offers the following room accommodations:

For one person, \$3.25 to \$6.00 per day
Double bed, for two persons, \$5.00 to \$6.50 per day
Twin beds, for two persons, \$5.50 to \$9.00 per day
Parlor Suites, single or double, \$20.00 per day

Welcome to Indianapolis

The local hospitality committee for the Indianapolis convention is planning entertainment for members of your family who plan to accompany you to Indianapolis but do not care to attend meetings. For further information they may inquire at the *Information Table.*